Constraints on the Size of Extra Compactified Dimensions from Compact Star Observations

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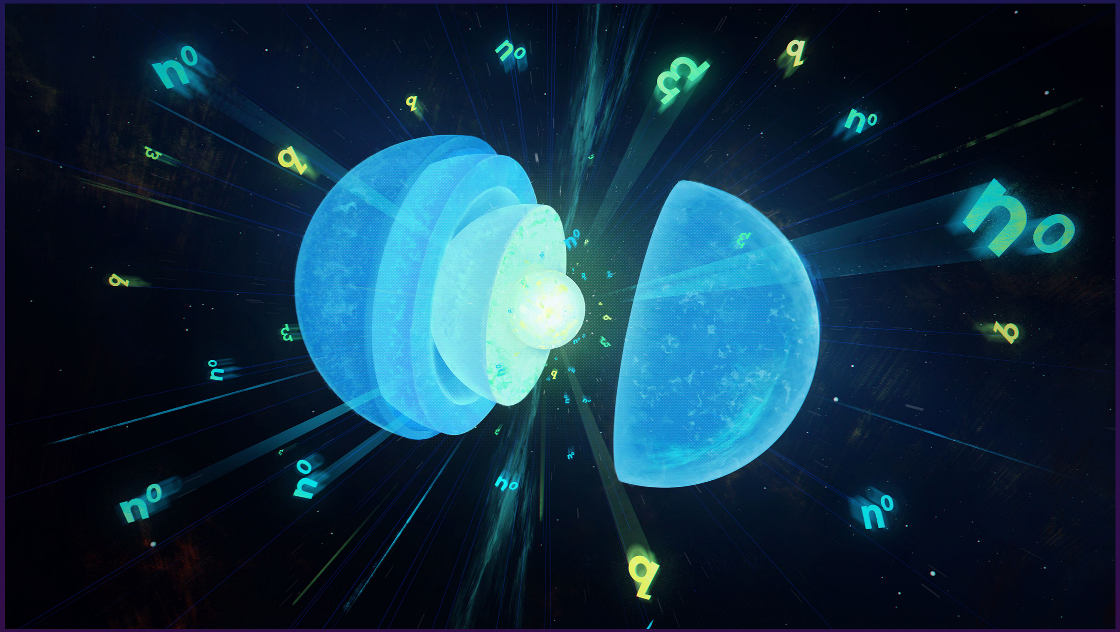

In collaboration with:
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Wigner Research Centre for Physics
Emese Forgács-Dajka
Eötvös Loránd University

Dark Matter and Stars 2023

Support: NKFIH OTKA K135515

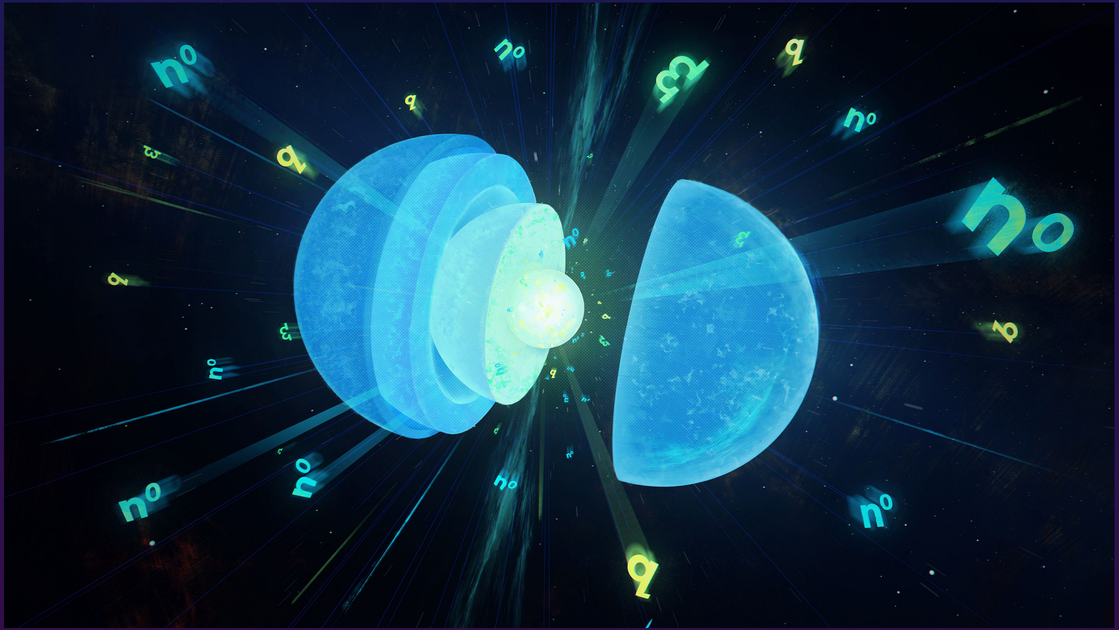
Constraints on the Size of Extra Compactified Dimensions from Compact Star Observations

- Remnants of supernovae
- Supported by baryon degeneracy
- Compact objects:

  $R \sim 10\text{km} \qquad M \sim 1.1\text{–}2M_\odot$
- High density, low temperature

↓

Probe physics in environments not available on Earth

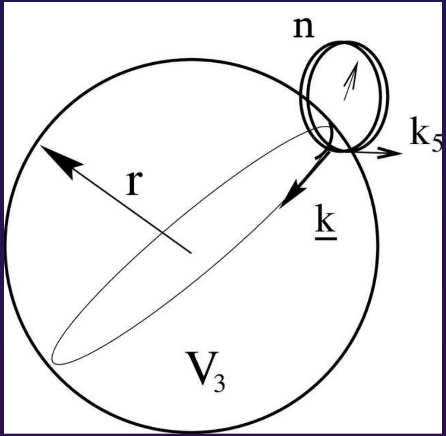


https://www.quantamagazine.org/squishy-or-solid-a-neutron-stars-insides-open-to-debate-20171030/

Treatment in $1+3+1_c$ dimensions

- Assume one extra **compactified spatial dimension** with size R_C
- At **each point** in ordinary 3D space **particles** with enough energy **can move into** it
  - 3D: particles with **different masses**
  - $3+1_c$D: **one particle** but with **different quantized momenta** in the extra dimension

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_C}\right)^2 + m^2} = \sqrt{\underline{k}^2 + \bar{m}^2}$$



- With the **right** choice of R_C the **mass spectrum** of particles could be **reproduced**

Torres del Castillo, Gerardo. (2019). An introduction to the Kaluza-Klein formulation.

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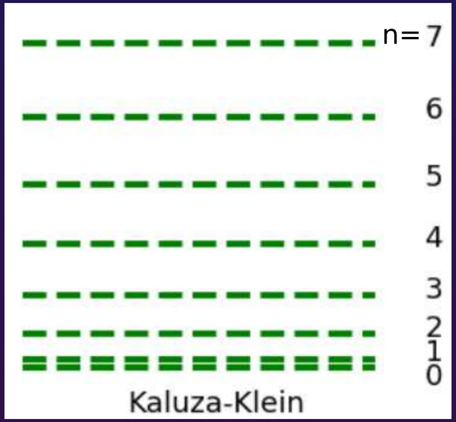
$$\bar{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$



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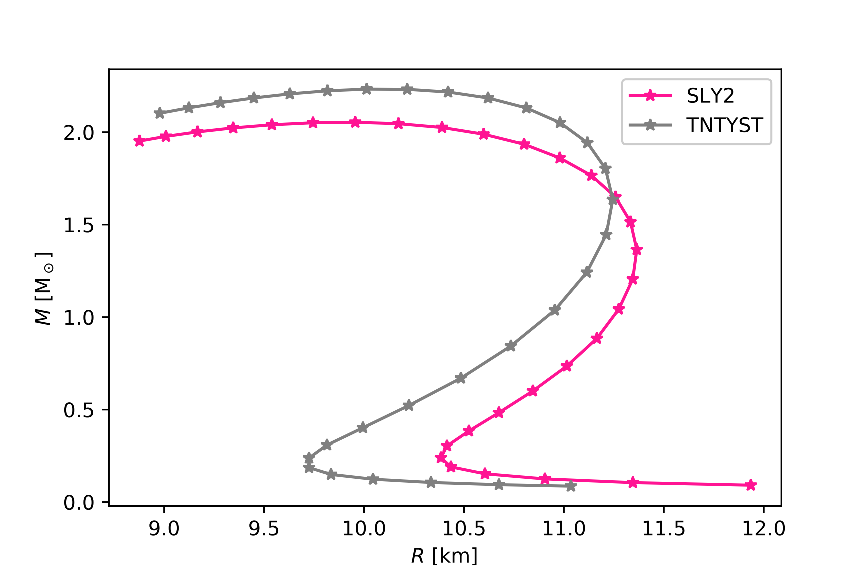
Building up stars

- Two equations needed:
  - Tolmann-Oppenheimer-Volkoff (TOV) ~ GR hydrostatic equation

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{GM(r)}{r}\right]^{-1}$$

static, spherically symmetric

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$



  - Equation of state (EoS) $\varepsilon(p)$
- Boundary conditions:
  - Pressure at the surface $p(R) = 0$

    (in practice $p(R) = p_{\mathrm{min}} = 10^{-5}\ \mathrm{km}^{-2}$)
  - Central energy density ε_C

↓

M-R diagrams

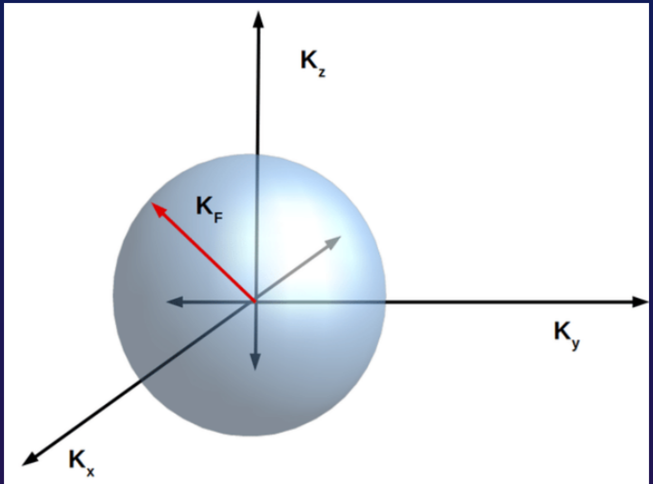
EoS in 1+4D

- **Interacting** degenerate **Fermi gas**
- **Potential** is a **linear** function of density:

  $U(n) = \xi n \qquad \xi = \text{const}$

- **Thermodynamic potential** on T=0 MeV

$$\widetilde{\Omega} = \frac{-2k_B T V_{(d)}}{h^d} \int \ln\left(1 + e^{\frac{\mu - E(\mathbf{p})}{k_B T}}\right) \mathrm{d}^d\mathbf{p}$$



- Extra dimension → calculate with **excited mass**
- Interaction → **chemical potential shifted** by $-U(n)$

$$\epsilon(\mu) = \epsilon_0(\mu - U(n)) + \epsilon_{int}$$

$$p(\mu) = p_0(\mu - U(n)) + p_{int}$$

$$n(\mu) = n_0(\mu - U(n))$$

$$\epsilon_{int} = p_{int} = \int U(n)dn = \int \xi n dn = \frac{1}{2}\xi n^2$$

J. Zimanyi, B. Lukacs, P. Levai, J.P. Bondorf: „An Interpretable Family ofEquation of State for Dense Hadronic Matter”, Nucl.Phys. A484 (1988) 647

Relativity in 1+4D

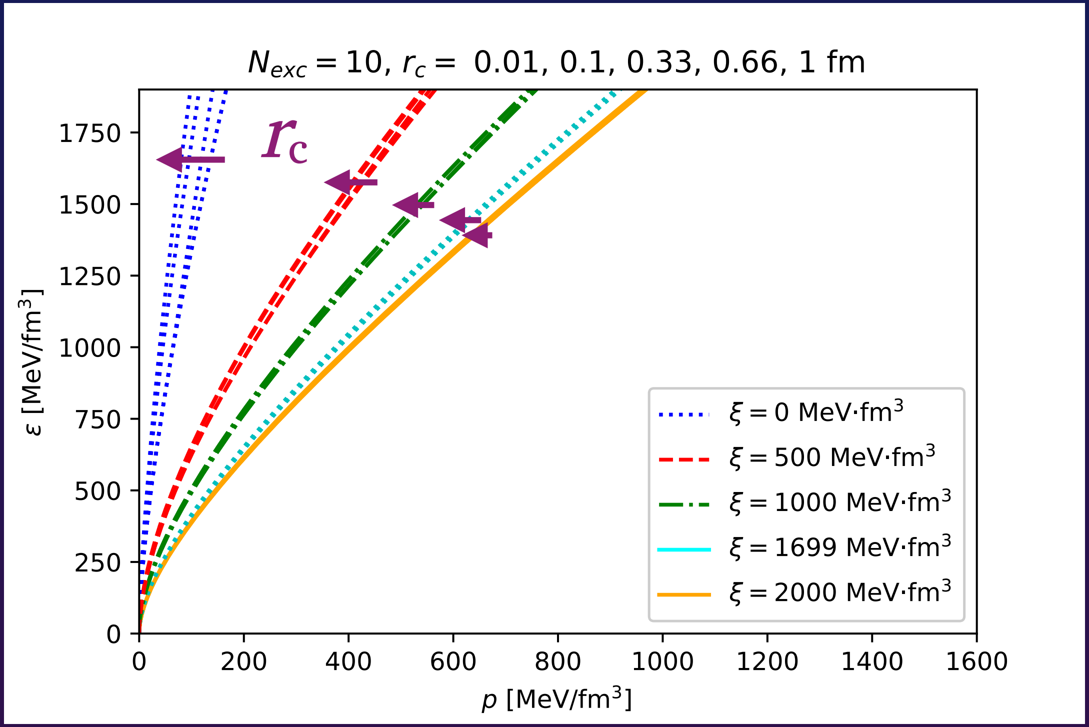
- Assume (for TOV):
  - Spherical symmetry
  - Time-independence
  - Isotropic relativistic ideal fluid
- Assume (for extra dimension):
  - Microscopic
  - 4D metric does not depend on g_{55}
  - Causality postulates hold
  - Full Killing symmetry

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & \cancel{g_{01}} & 0 & 0 & \cancel{g_{05}} \\ \cancel{g_{01}} & g_{11} & 0 & 0 & \cancel{g_{15}} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ \cancel{g_{05}} & \cancel{g_{15}} & 0 & 0 & \boxed{g_{55}} \end{bmatrix}$$

G. G. Barnaföldi, P. Lévai, B.Lukács et al. Astron. Nachr. 328,809 (2007)

Equation of state

- ξ **dependence** is much more **dominant** than r_c
- The **bigger** ξ, the **less important** r_c becomes



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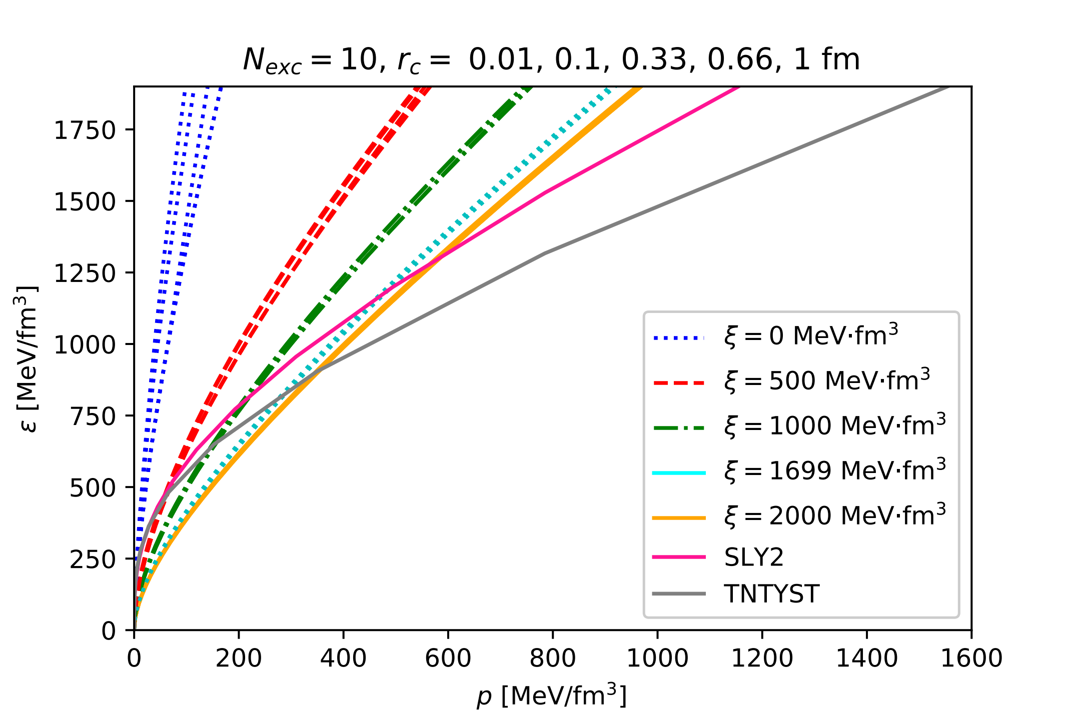
- For **lower energies small** ξ **approximates** more refined nuclear matter EoSs
- For **high energies** a **large** ξ is a better approximation



https://compose.obspm.fr/
1. H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki and M. Takano, Nucl. Phys. A 961 (2017) 78
1. F. Gulminelli and Ad. R. Raduta, arXiv:1504.04493.
2. E. Chabanat, Ph.D. thesis, University Claude Bernard Lyon-1, Lyon, France, 1995.
3. P. Danielewicz et J. Lee, Nucl. Phys. A818, 36 (2009).

M-R diagrams of the EoS

- ξ **dependence** is much more **dominant** than r_c (latter only ∼5%)
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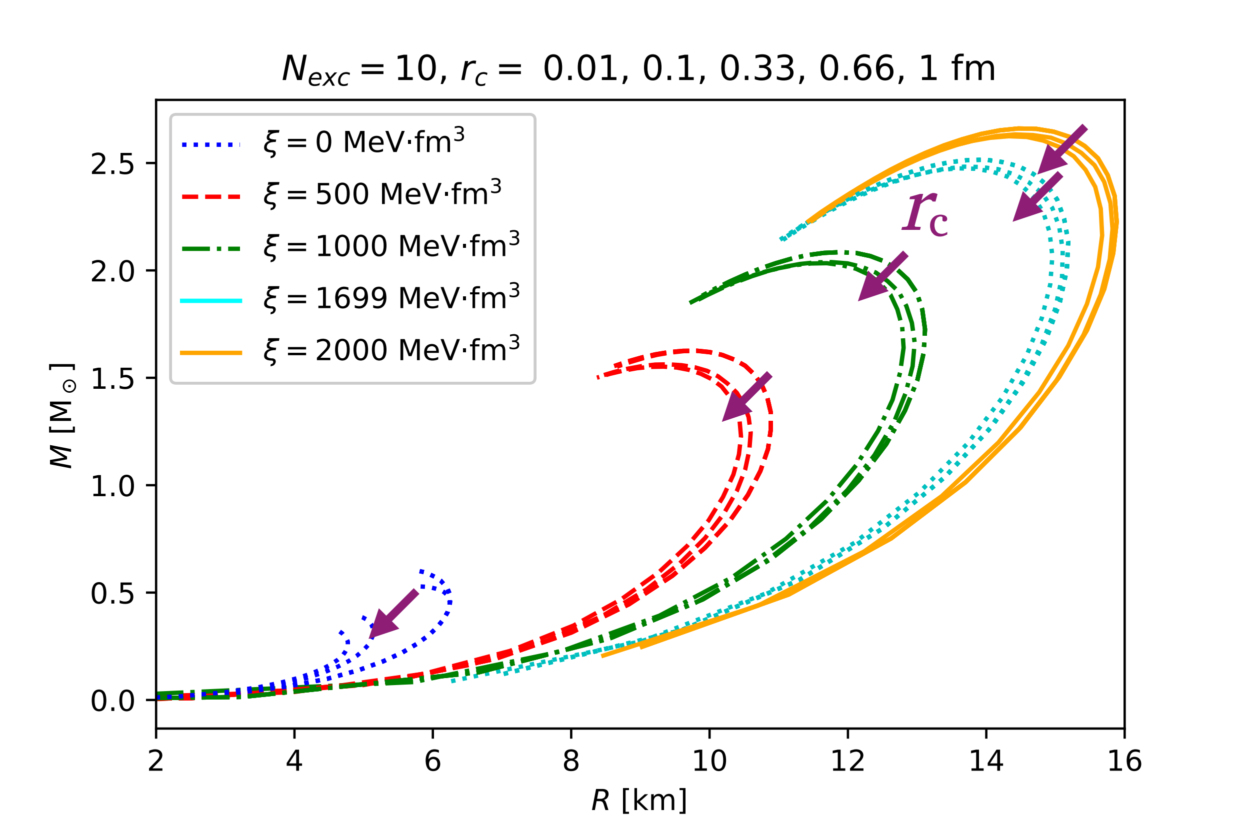

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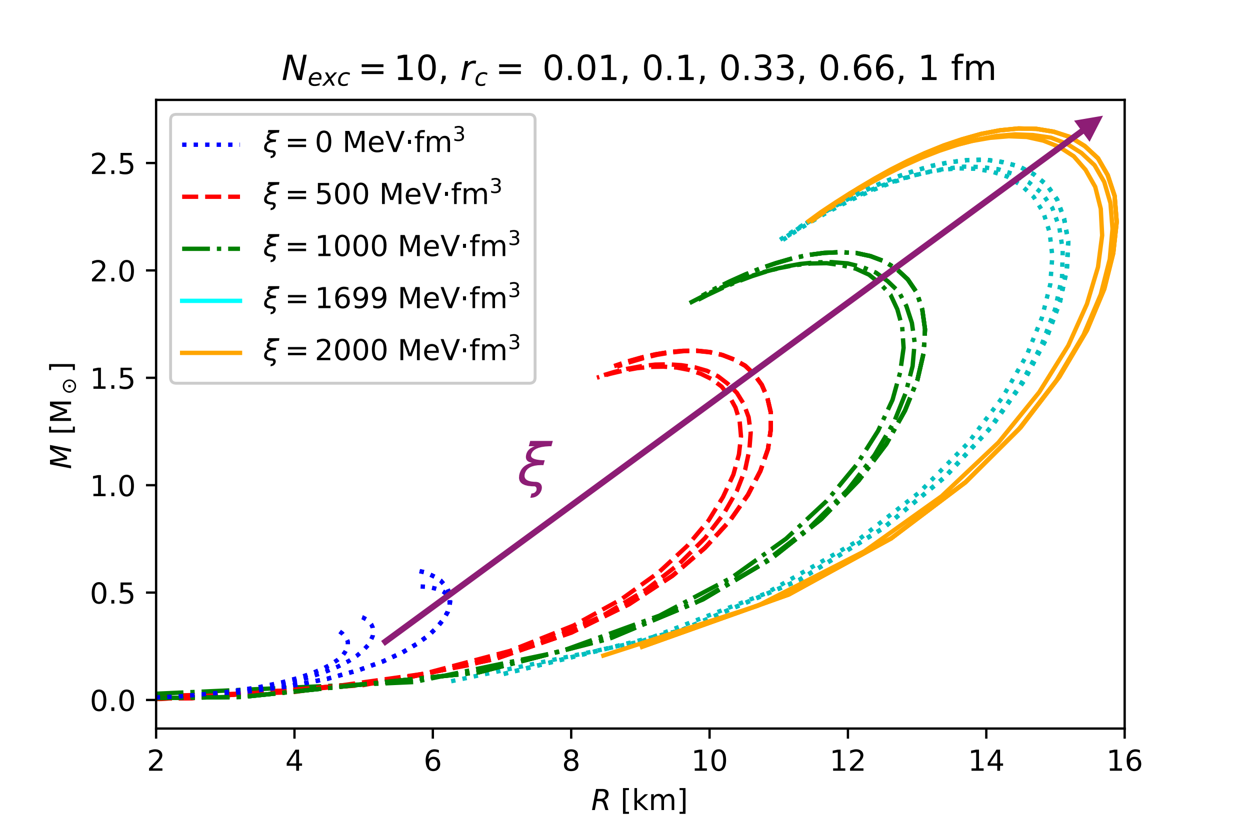
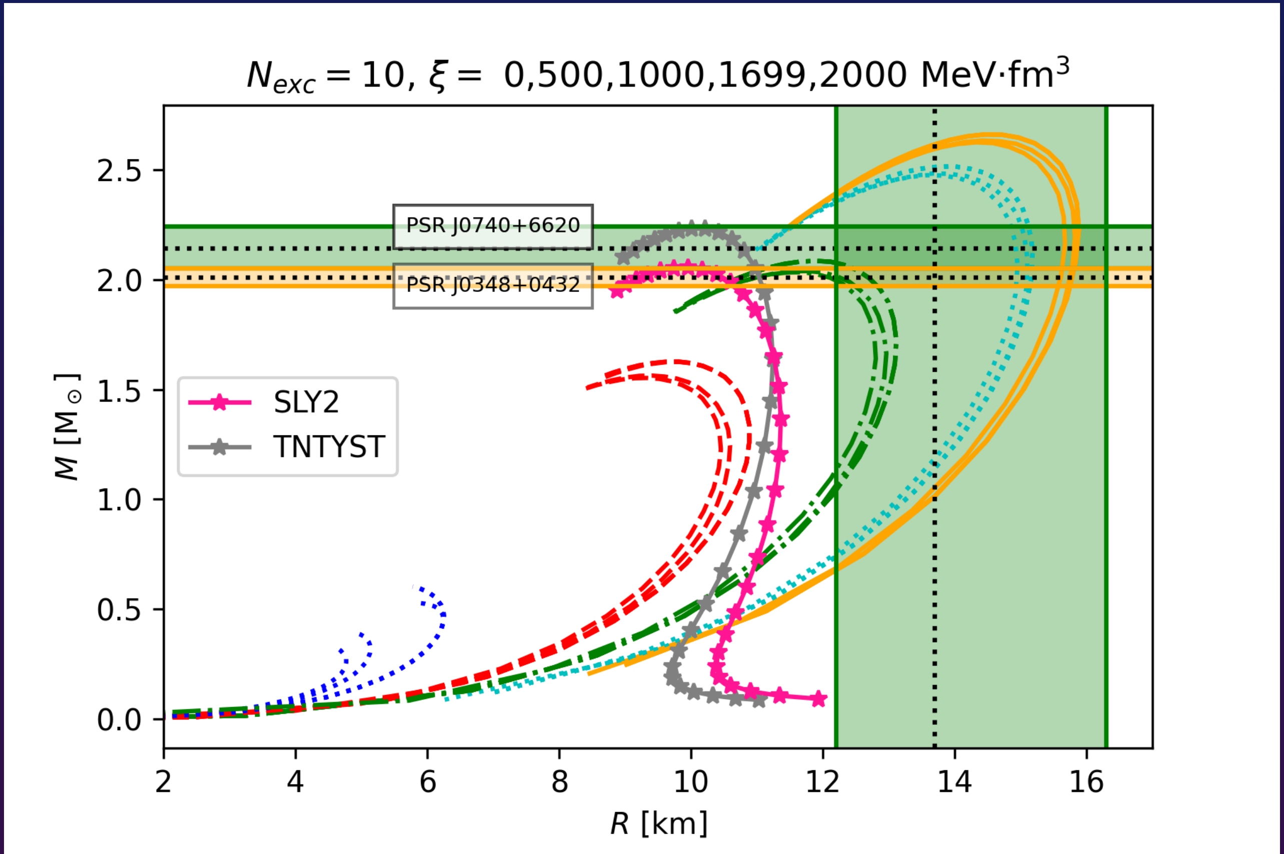


M-R diagrams

+ measurement data

+ 2 more refined EoSs



M. C. Miller et al 2021 ApJL 918 L28
H.T. Cromartie et al., Nat. Astron. 4, 72 (2019)
J. Antoniadis et al., Science 340, 1233232 (2013)

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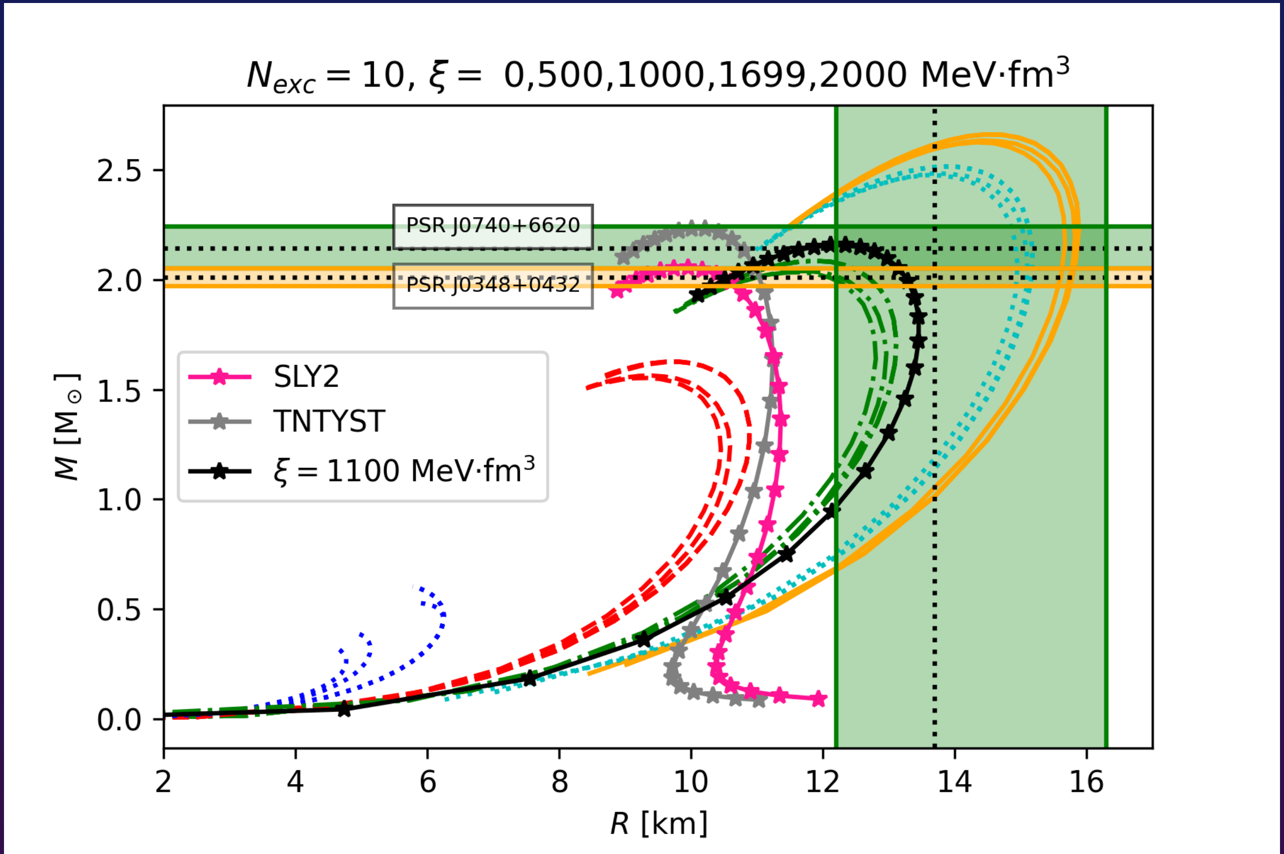

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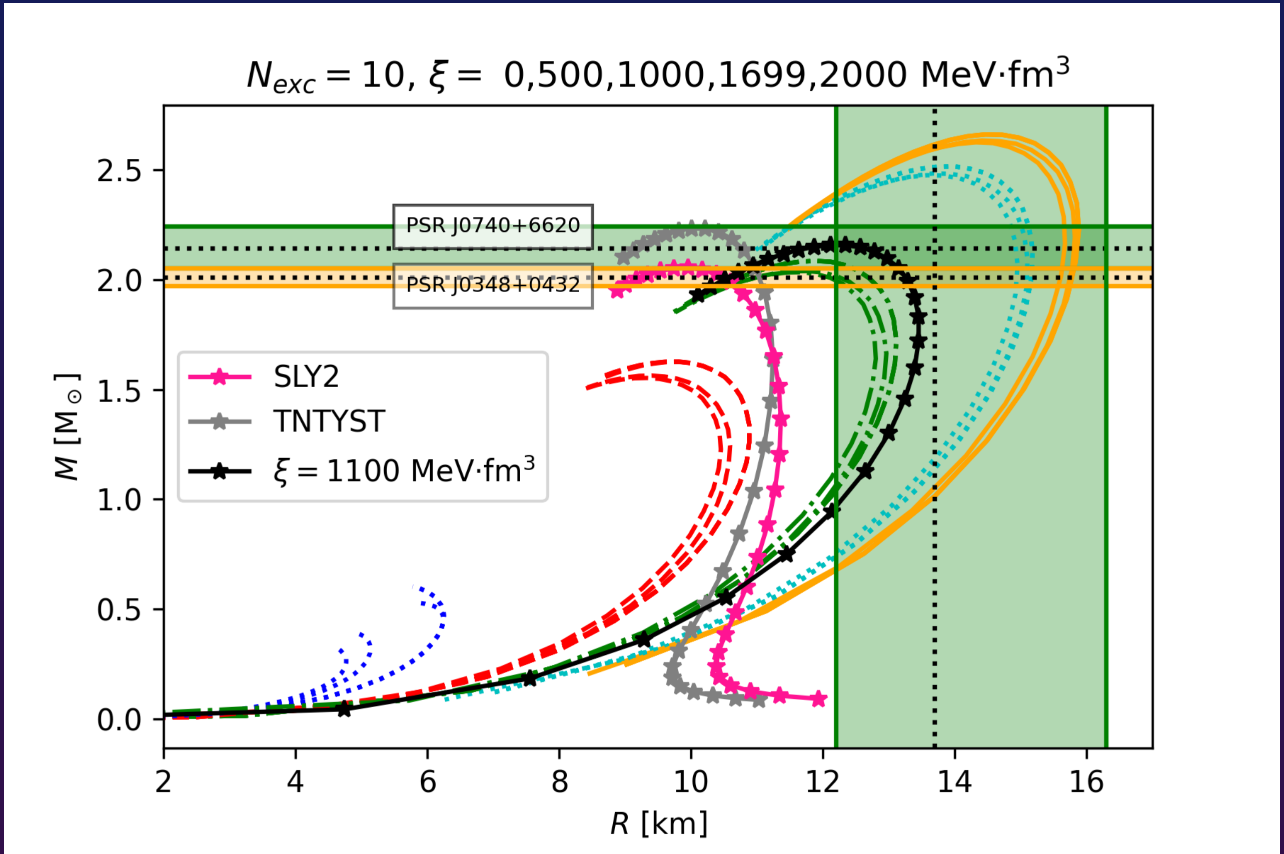

Change position of black curve by **setting** the **size of** the **extra dimension**.

M. C. Miller et al 2021 ApJL 918 L28
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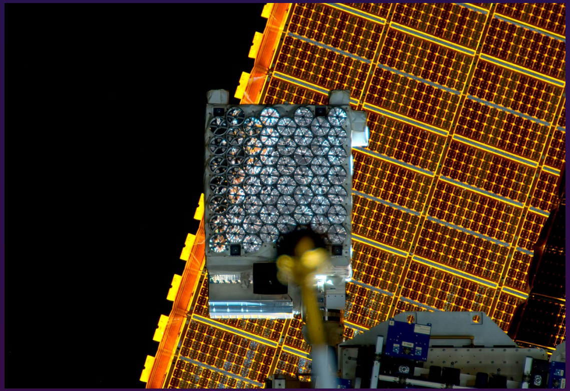
Summary

LIGO



NICER

- Model with the possibility of probing **beyond standard model physics**
- One **extra** spatial **compactified dimension**

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Ordinary **mass** can be described as **quantized 5^{th}D momenta**

- Effective nuclear field theory with **linear** repulsive **potential**

- It is **possible** to **build** compact stars with **realistic** properties
- **Constraints** on the **size of** possible **extra dimensions** could be given using more precise **observational data**

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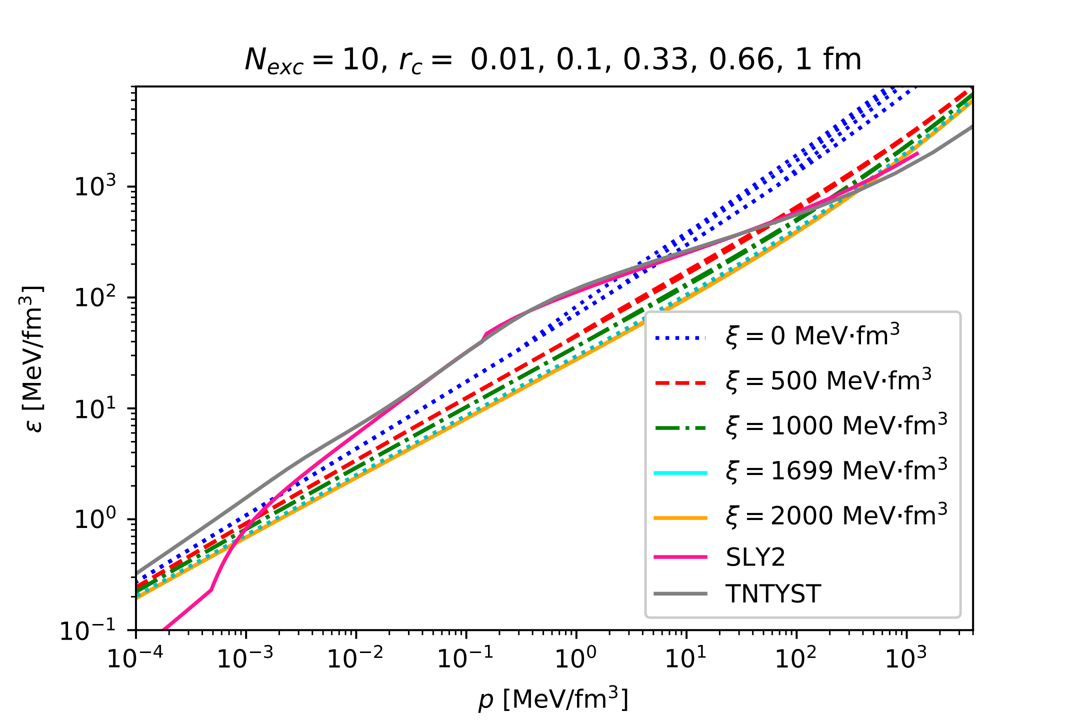

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