Neutron stars in 1+4D with interacting Fermi gas

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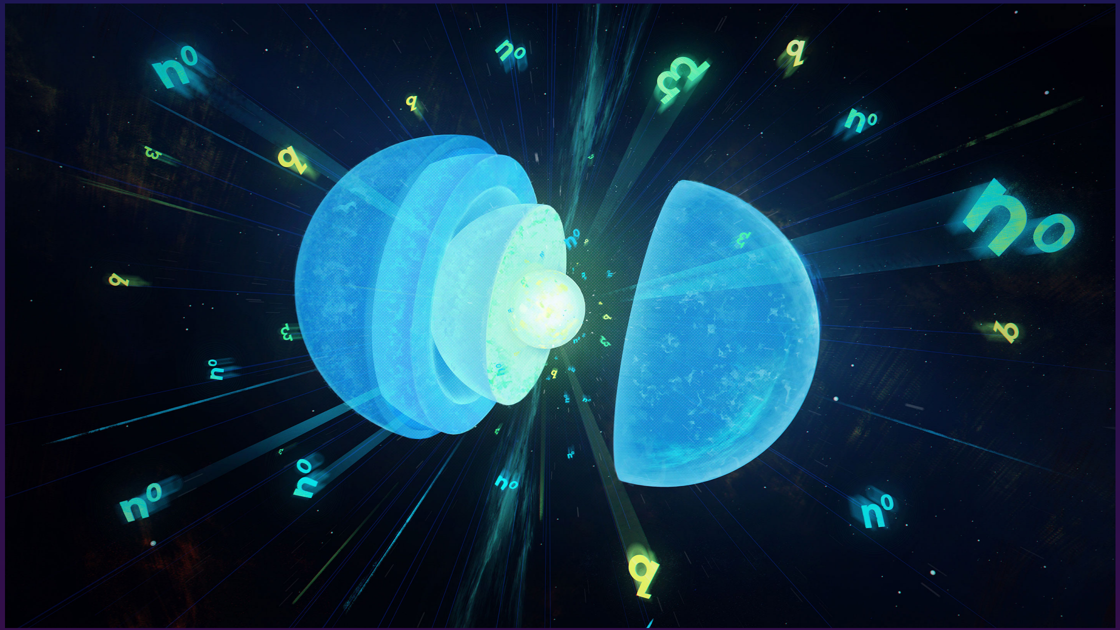
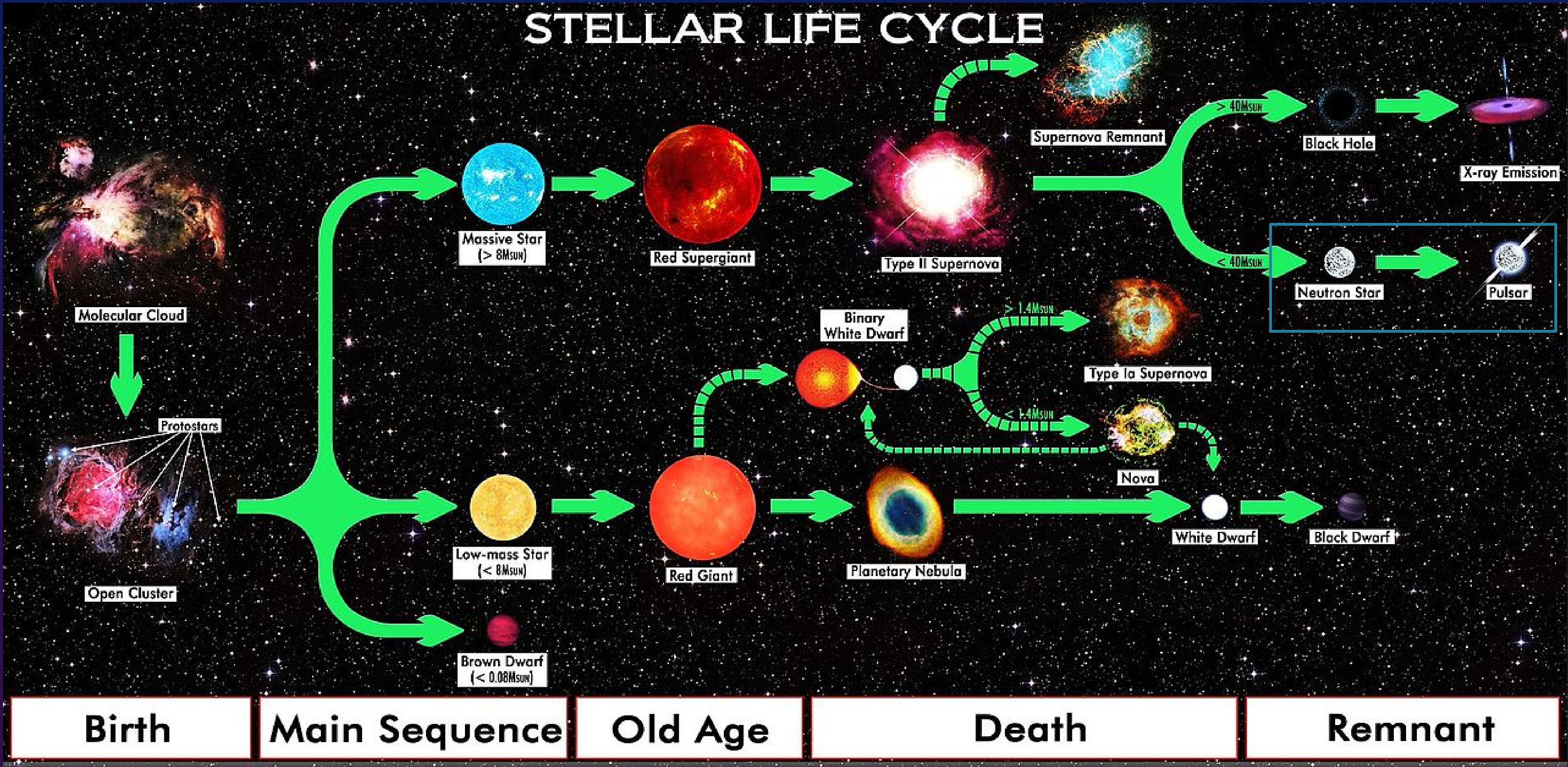


ELTE AstroPizza 2023

Support: NKFIH OTKA K135515



https://en.wikipedia.org/wiki/Black_dwarf

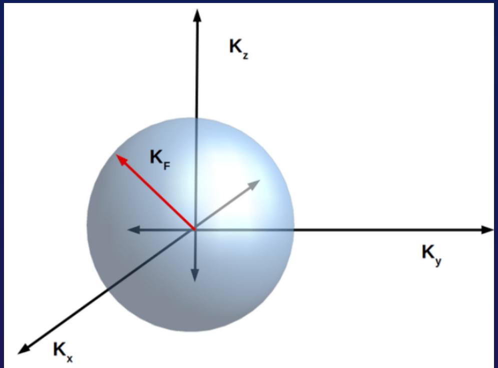
Neutron stars

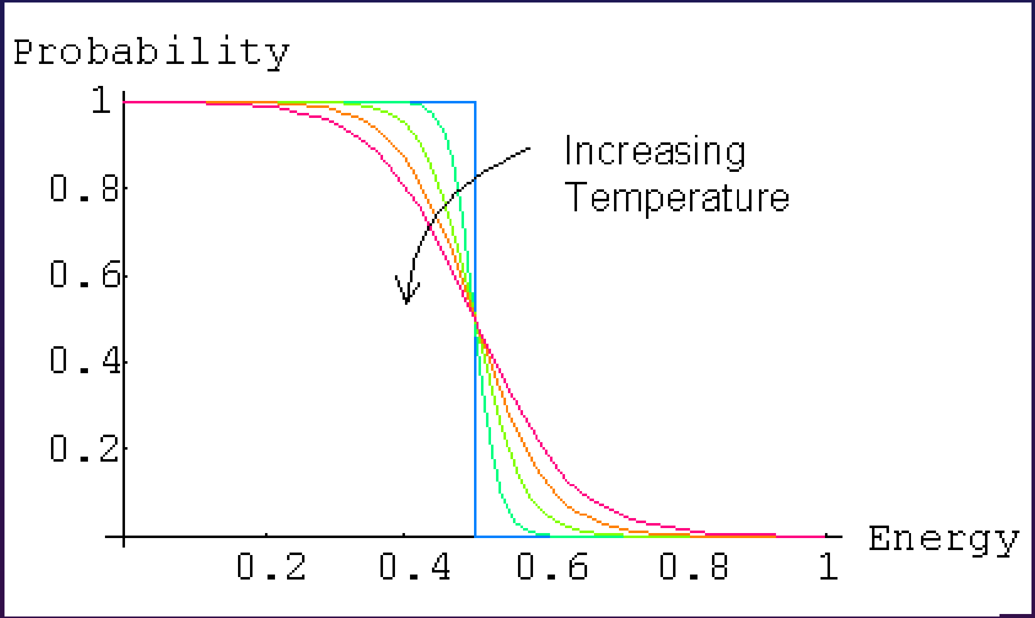
- Compact objects:

  $R \sim 10\text{km} \qquad M \sim 1.1\text{–}2M_\odot$
- Supported by baryon degeneracy
  – Pauli exclusion principle
- Fermi-Dirac distribution

$$\bar{n}_i = \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} + 1}$$

- Zero temperature approximation





Fermi-Dirac distribution

QCD phase daigram

- Collider experiments probe high temperature, low density regime (RHIC, LHC)
- Neutron stars reside in the high density, low temperature regime

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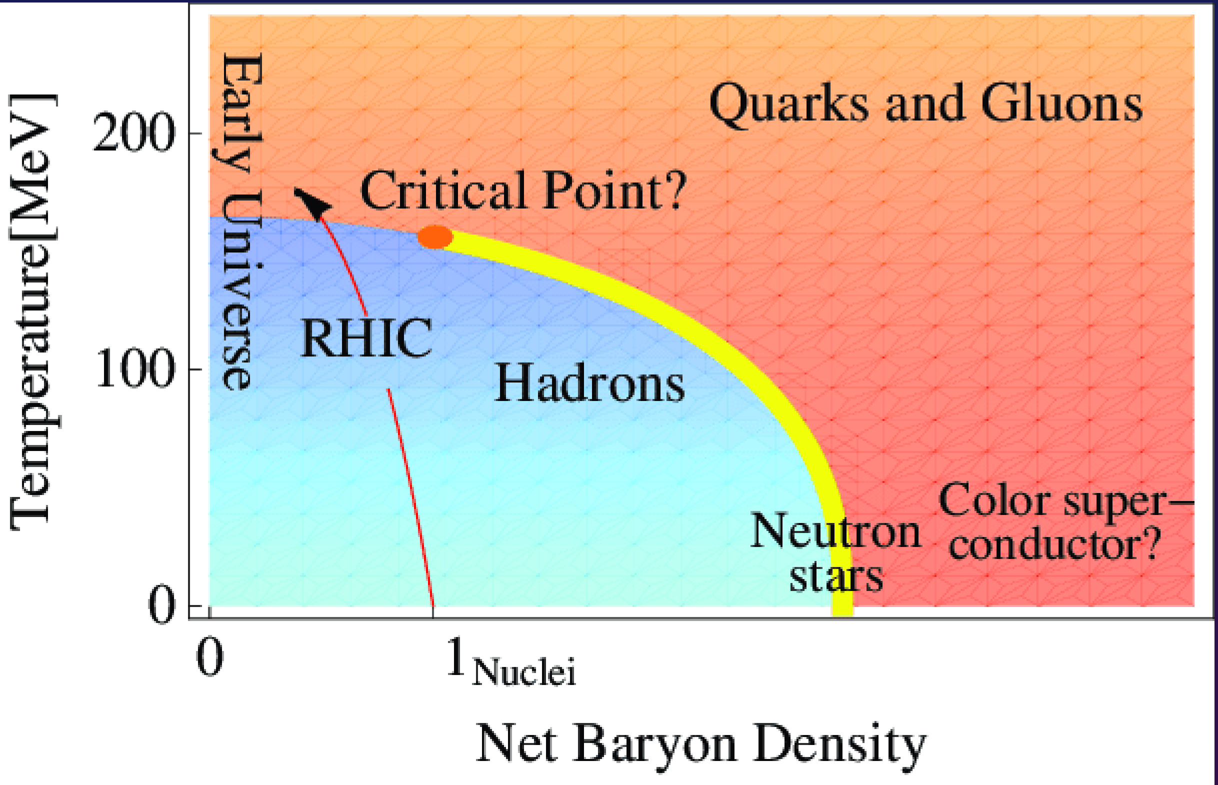
Probe physics in environments not available on Earth

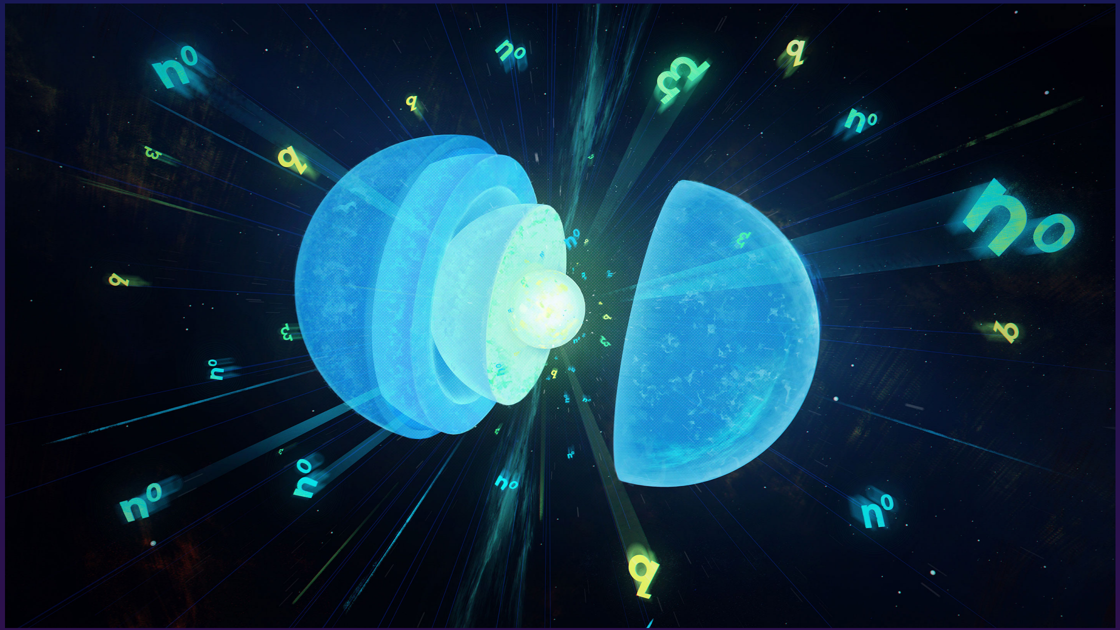


Probe physics of neutron stars

- Compare numerical simulations with experimental data – building up stars
- Main observables:

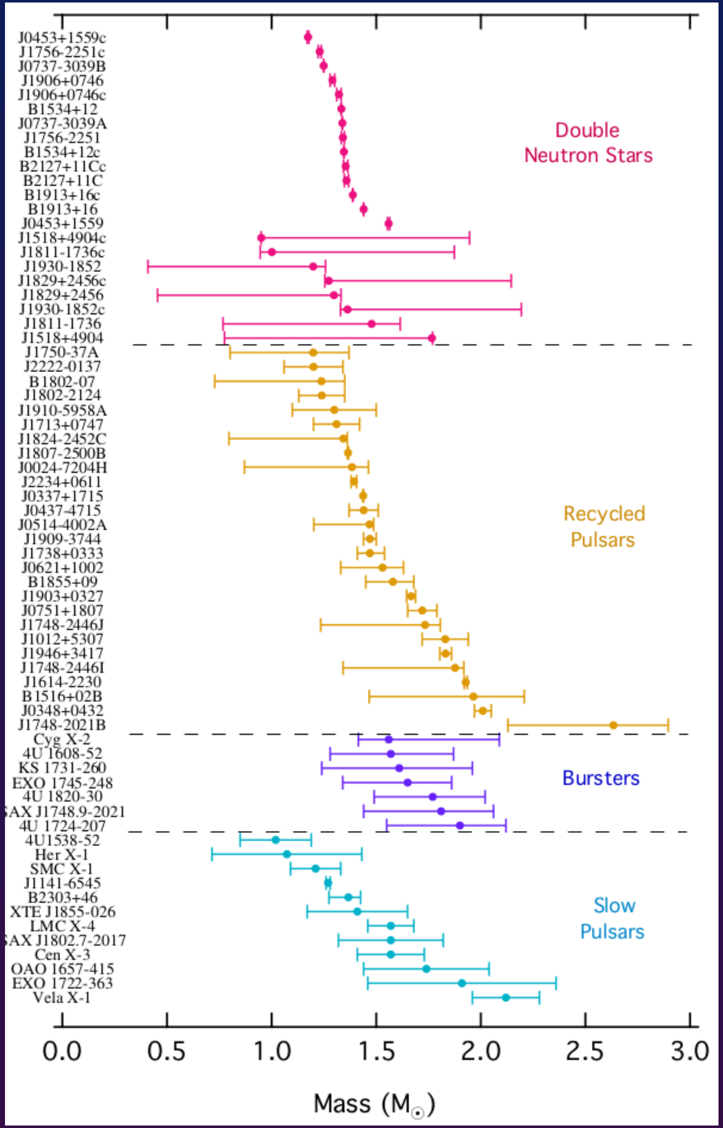
  mass – difficult to measure
  - Error ~ 2–5%

  radius – even more difficult
  - Error ~ 10–15%



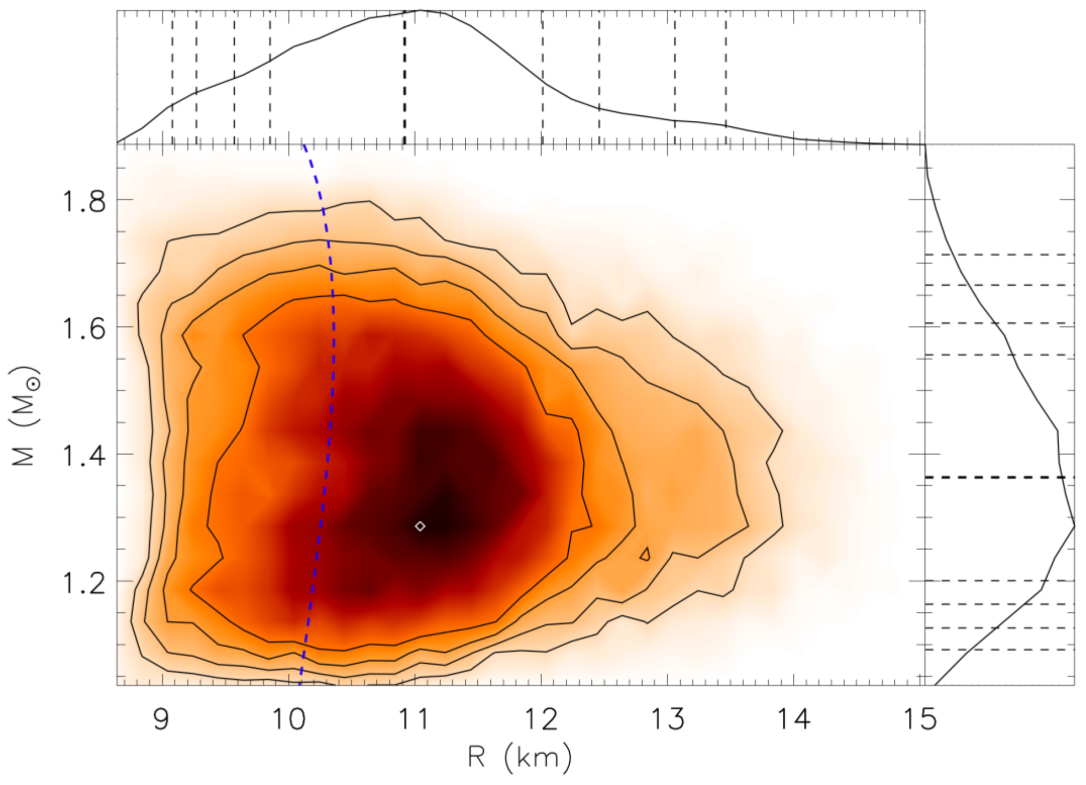
Mass measurements

- Radio measurement of **rotation powered pulsars** – most precise
- Also: X-rays, gamma rays, gravitational waves
- **Binary** or multiple component **systems** – tracking of **orbital motion**
- Better if both compact – point masses
- Isolated cannot be measured!
- **Methods**:
  - Advance of periastron
  - Einstein delay
  - Orbital period decay
  - Shapiro delay



Radius measurements

- **Sources**:
    - Nuclear physics
    - Astrophysical observations
    - GR assumptions
    - Causality of EoS
- Measurements based on **thermal emission** from surface:
    - Apparent angular size
    - Effects of NS spacetime on emission
    - **Spectroscopy** and **timing**
- **Gravitational waves** (figure: GW170817)



James M. Lattimer, "Neutron Star Mass and Radius Measurements" (2019)

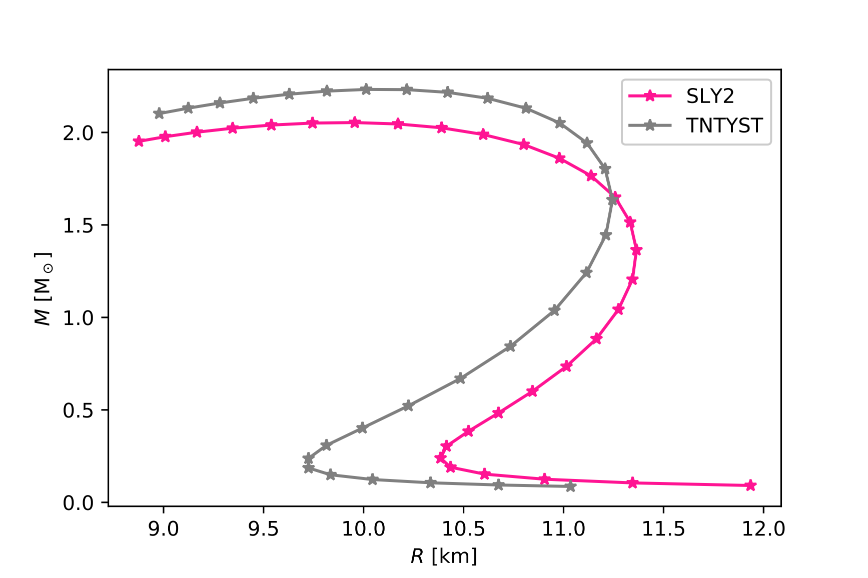
Building up stars

- Two equations needed:
  - Tolmann-Oppenheimer-Volkoff (TOV) ~ GR hydrostatic equation

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1 + \frac{p(r)}{\varepsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{M(r)}\right]\left[1 - \frac{GM(r)}{r}\right]^{-1}$$

static, spherical symmetry

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$



  - Equation of state (EoS) $\varepsilon(p)$
- Boundary conditions:
  - Pressure at the surface $p(R) = 0$
    (in practice $p(R) = p_{\mathrm{min}} = 10^{-5}$ km^{-2})
  - Central energy density ε_C

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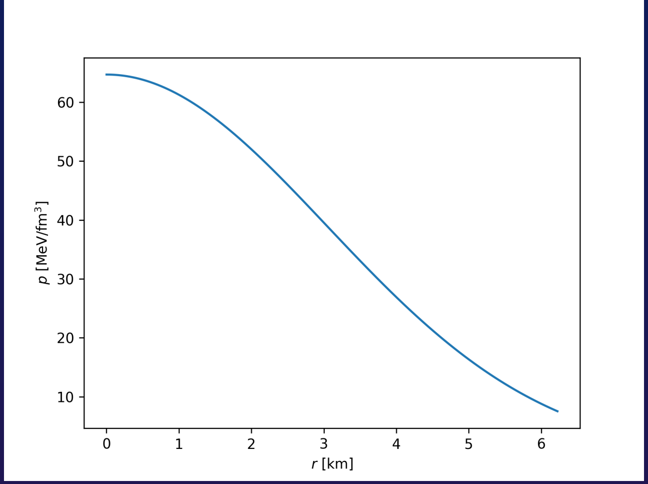
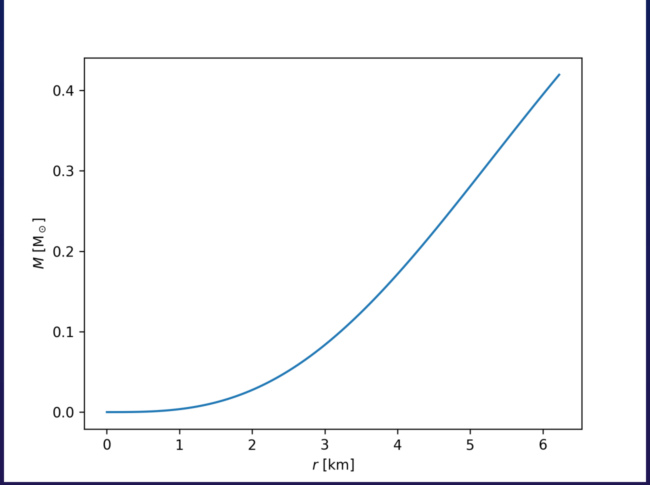
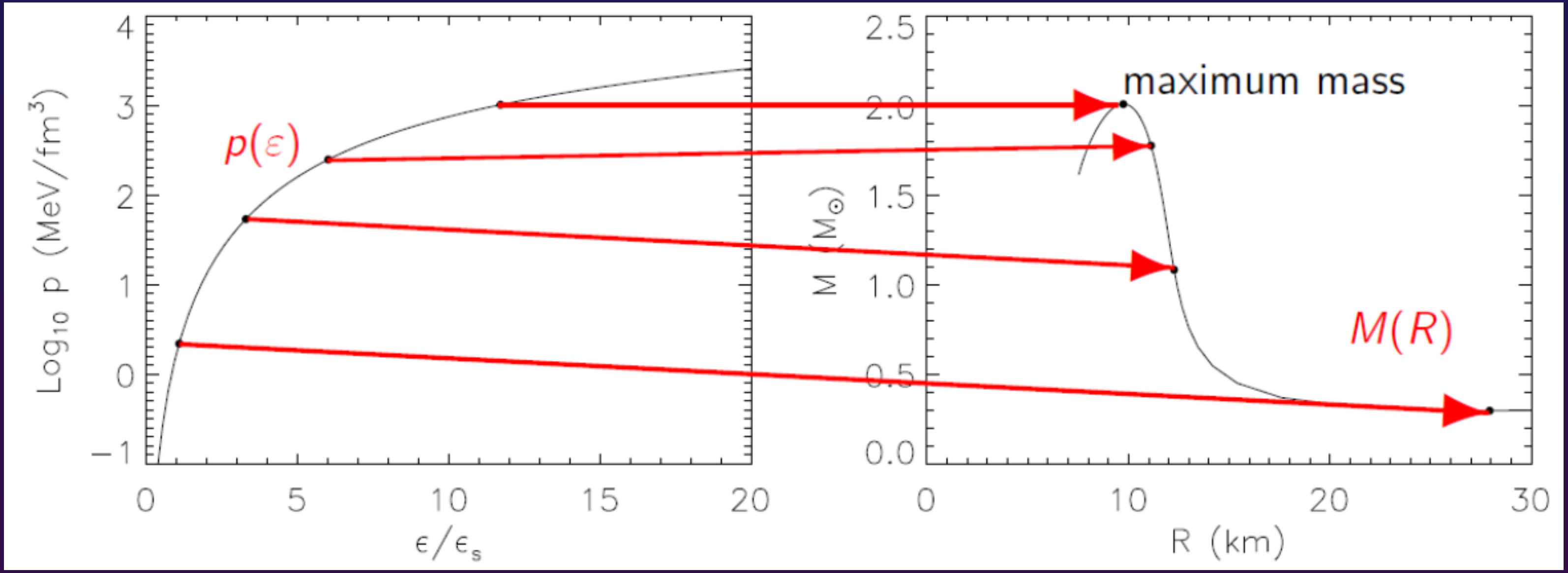
M-R diagrams







G. G. Barnaföldi MTA Pulzár 50 (2017)

Kaluza-Klein theory

- Originally: unified field theory of gravity and EM
- One extra curled up (compactified) dimension
- Geometrization of forces, e.g.:
  - Newtonian gravity – gravity is a force
  - General relativity – gravity arises from the curved geometry of spacetime
- We describe the mass spectrum of nucleons with the help of an extra dimension

Oskar Klein (1894-1977)

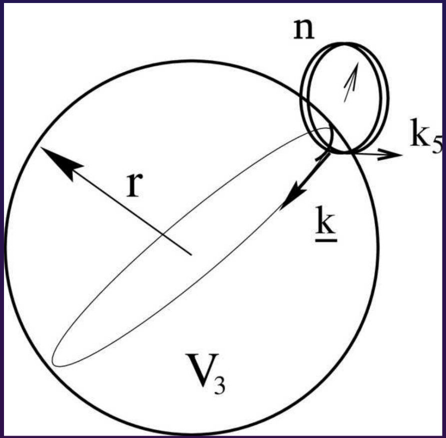
Theodor Kaluza (1885-1954)

Treatment in $1+3+1_c$ dimensions

- Assume one extra **compactified spatial dimension** with size R_C
- At **each point** in ordinary 3D space **particles** with enough energy **can move into** it
  - 3D: particles with **different masses**
  - $3+1_c$D: **one particle** but with **different quantized momenta** in the extra dimension

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_C}\right)^2 + m^2} = \sqrt{\underline{k}^2 + \bar{m}^2}$$

$$\bar{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$



- With the **right** choice of R_C the **mass spectrum** of particles could be **reproduced**

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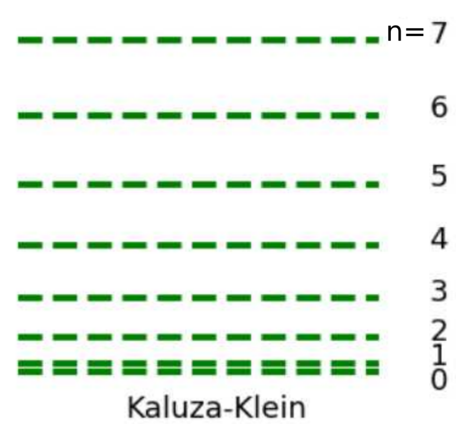

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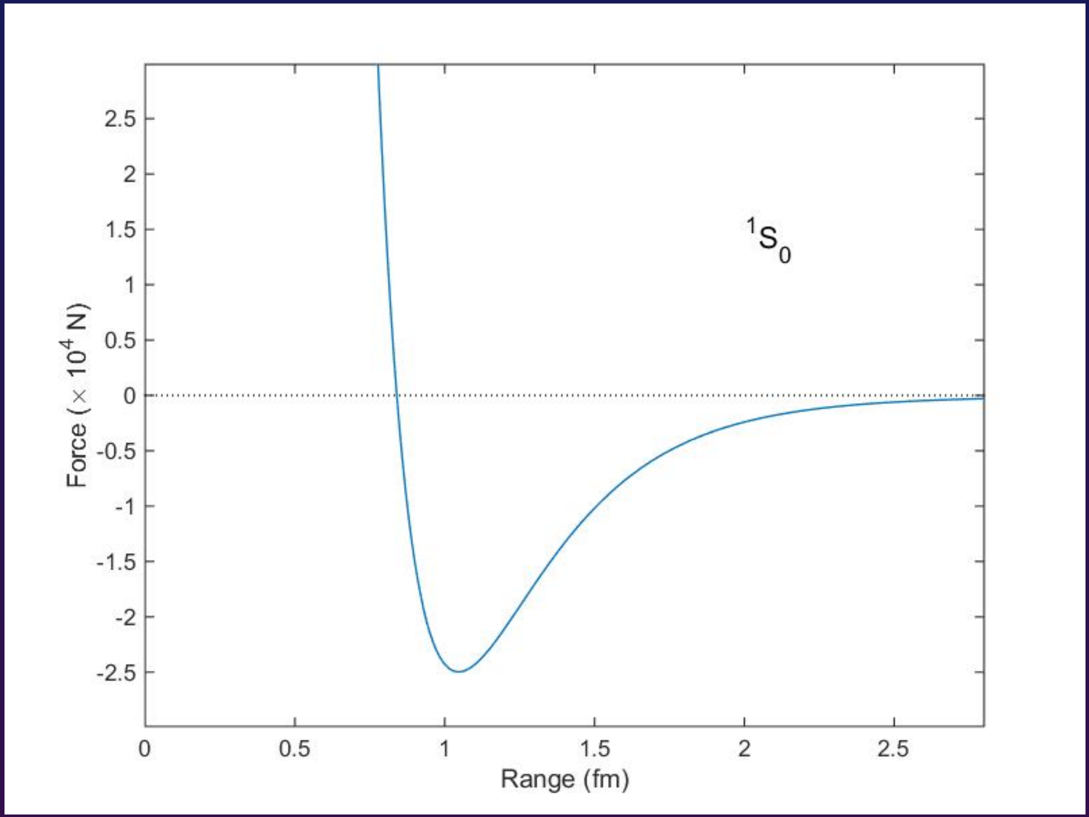
We also need interaction...

- Pauli exclusion principle is not the only thing that keeps particles apart
- We need to model the **strong force**:

  repulsive at this scale
- Without interaction the maximum mass of stars would become too small $\sim 0.7 M_{\odot}$



https://en.wikipedia.org/wiki/Nuclear_force

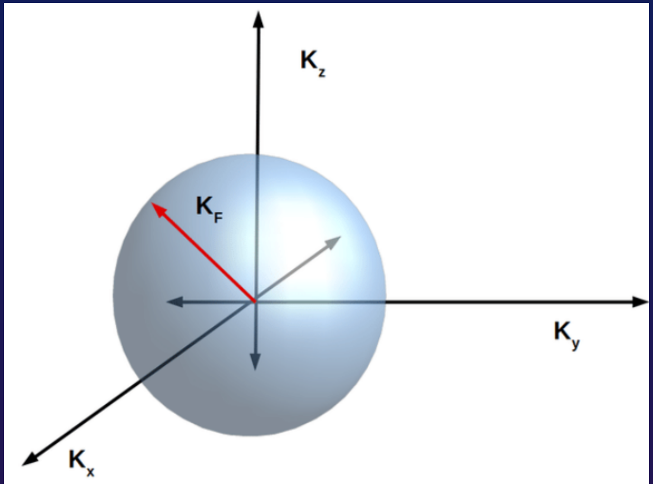
EoS in 1+4D

- **Interacting** degenerate **Fermi gas**
- **Potential** is a **linear** function of density:

  $U(n) = \xi n \qquad \xi = \text{const}$

- **Thermodynamic potential** on T=0 MeV

$$\widetilde{\Omega} = \frac{-2k_B T V_{(d)}}{h^d} \int \ln\left(1 + e^{\frac{\mu - E(\mathbf{p})}{k_B T}}\right) \mathrm{d}^d\mathbf{p}$$



- Extra dimension ⟶ calculate with **excited mass**
- Interaction ⟶ **chemical potential shifted** by $-U(n)$

$$\epsilon(\mu) = \epsilon_0(\mu - U(n)) + \epsilon_{int}$$

$$p(\mu) = p_0(\mu - U(n)) + p_{int}$$

$$n(\mu) = n_0(\mu - U(n))$$

$$\epsilon_{int} = p_{int} = \int U(n) dn = \int \xi n dn = \frac{1}{2}\xi n^2$$

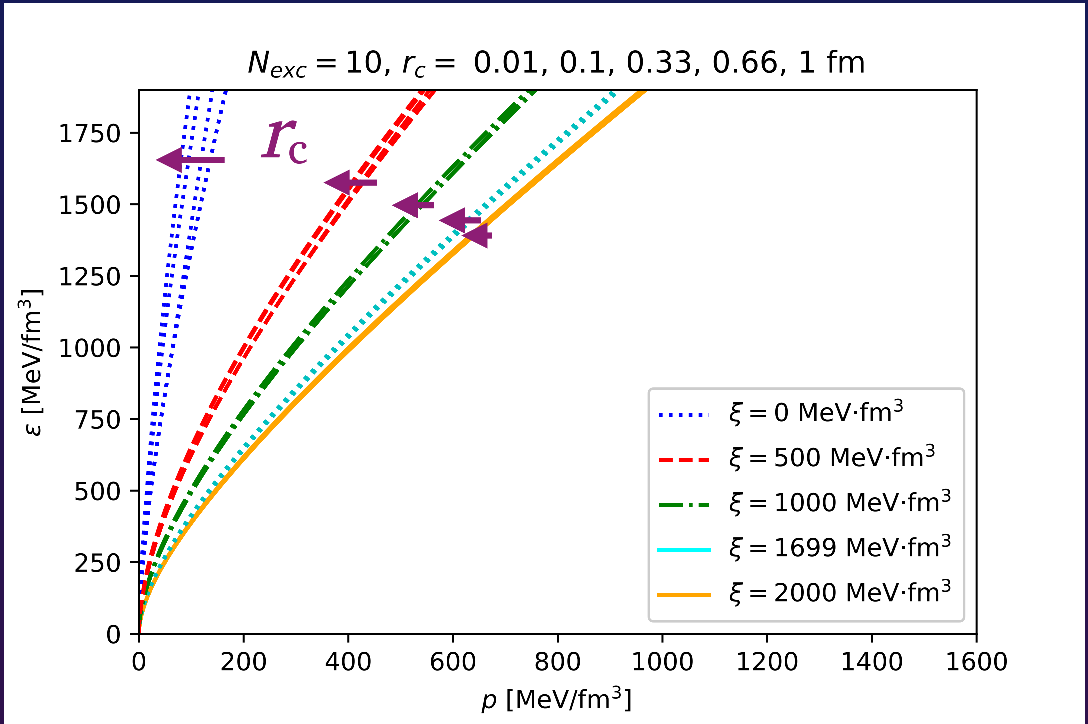
Relativity in 1+4D

- Assume (for TOV):
  - Spherical symmetry
  - Time-independence
  - Isotropic relativistic ideal fluid
- Assume (for extra dimension):
  - Microscopic
  - 4D metric does not depend on g_{55}
  - Causality postulates hold
  - Full Killing symmetry

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & \cancel{g_{01}} & 0 & 0 & \cancel{g_{05}} \\ \cancel{g_{01}} & g_{11} & 0 & 0 & \cancel{g_{15}} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ \cancel{g_{05}} & \cancel{g_{15}} & 0 & 0 & \boxed{g_{55}} \end{bmatrix}$$

Equation of state

- ξ **dependence** is much more **dominant** than r_c
- The **bigger** ξ, the **less important** r_c becomes



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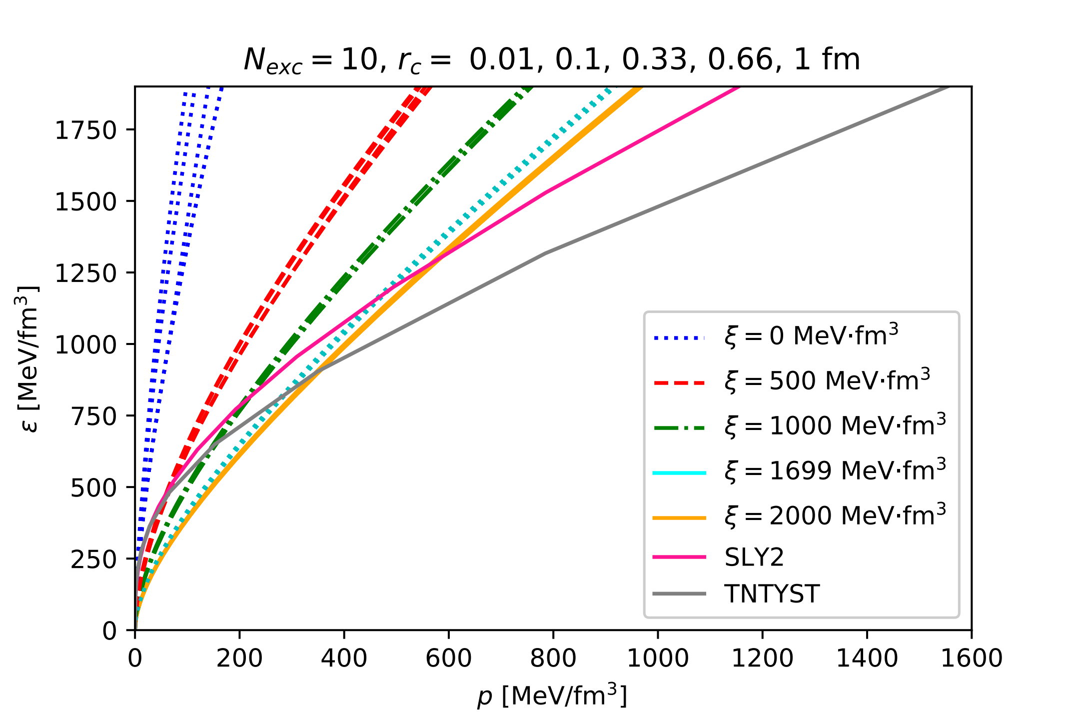
- For **lower energies small** ξ **approximates** more realistic EoSs
- For **high energies** a **large** ξ is a better approximation



https://compose.obspm.fr/
1. H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki and M. Takano, Nucl. Phys. A 961 (2017) 78
1. F. Gulminelli and Ad. R. Raduta, arXiv:1504.04493.
2. E. Chabanat, Ph.D. thesis, University Claude Bernard Lyon-1, Lyon, France, 1995.
3. P. Danielewicz et J. Lee, Nucl. Phys. A818, 36 (2009).

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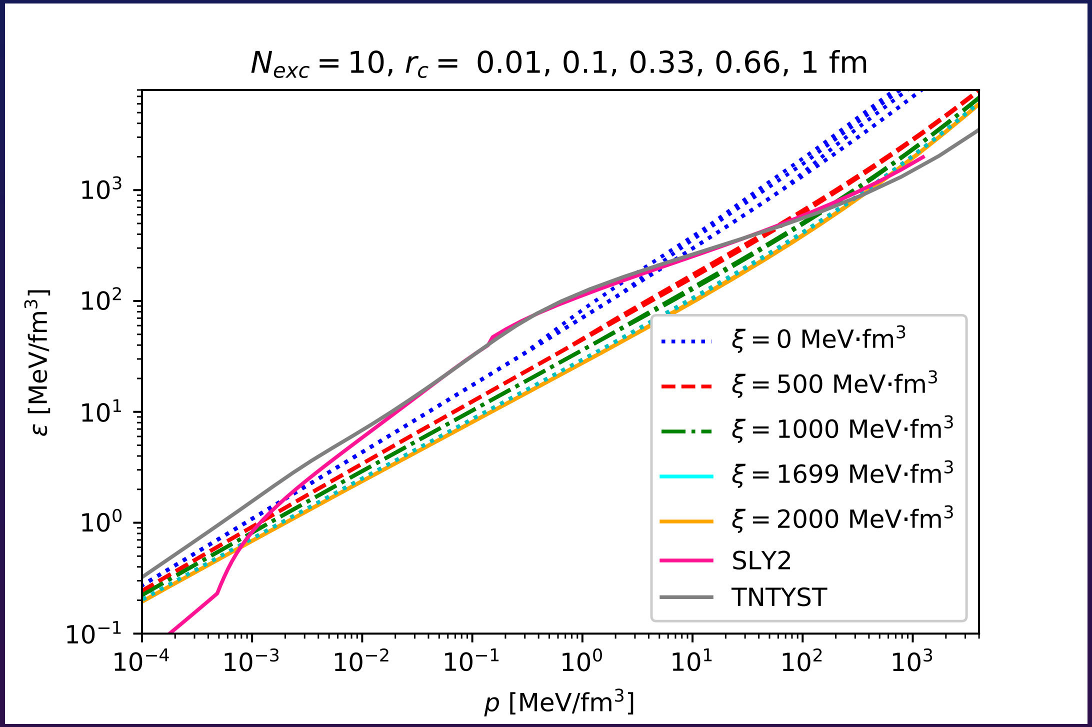

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M-R diagrams of the EoS

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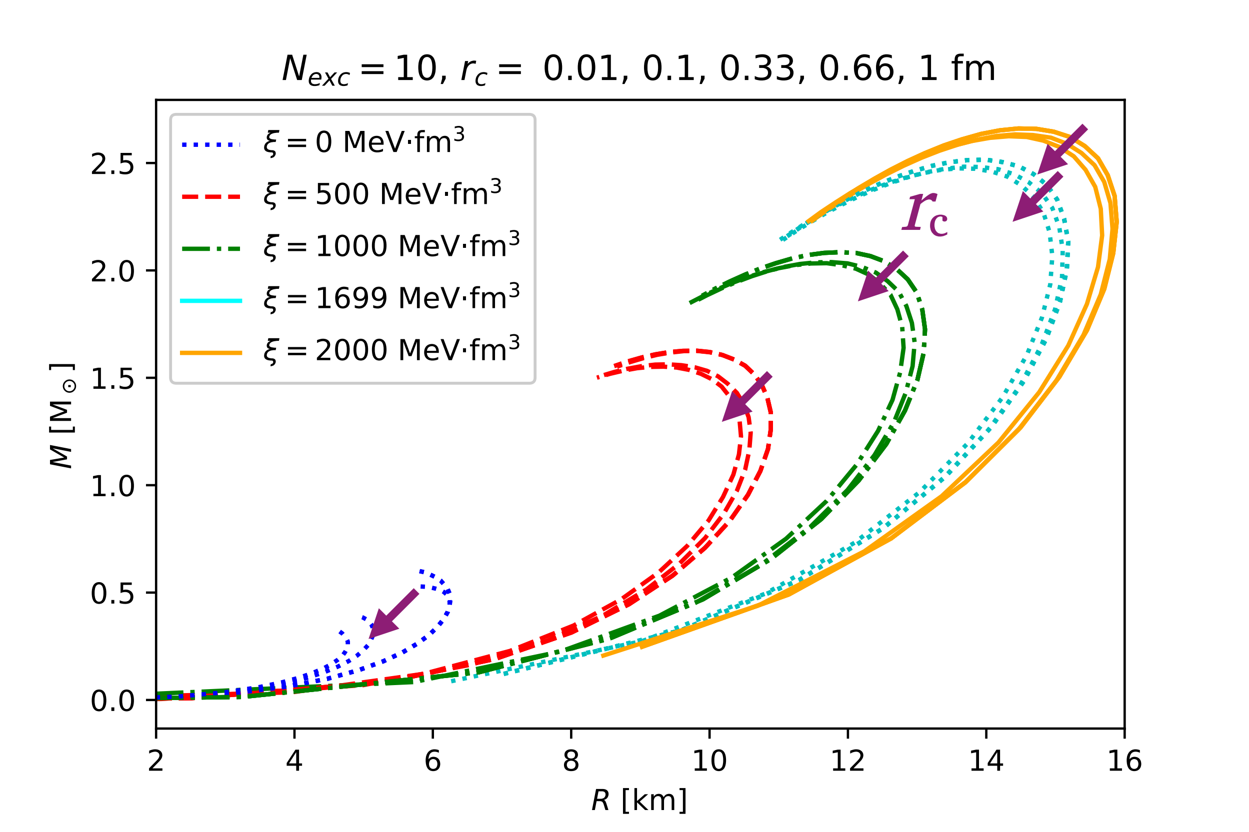

M-R diagrams of the EoS

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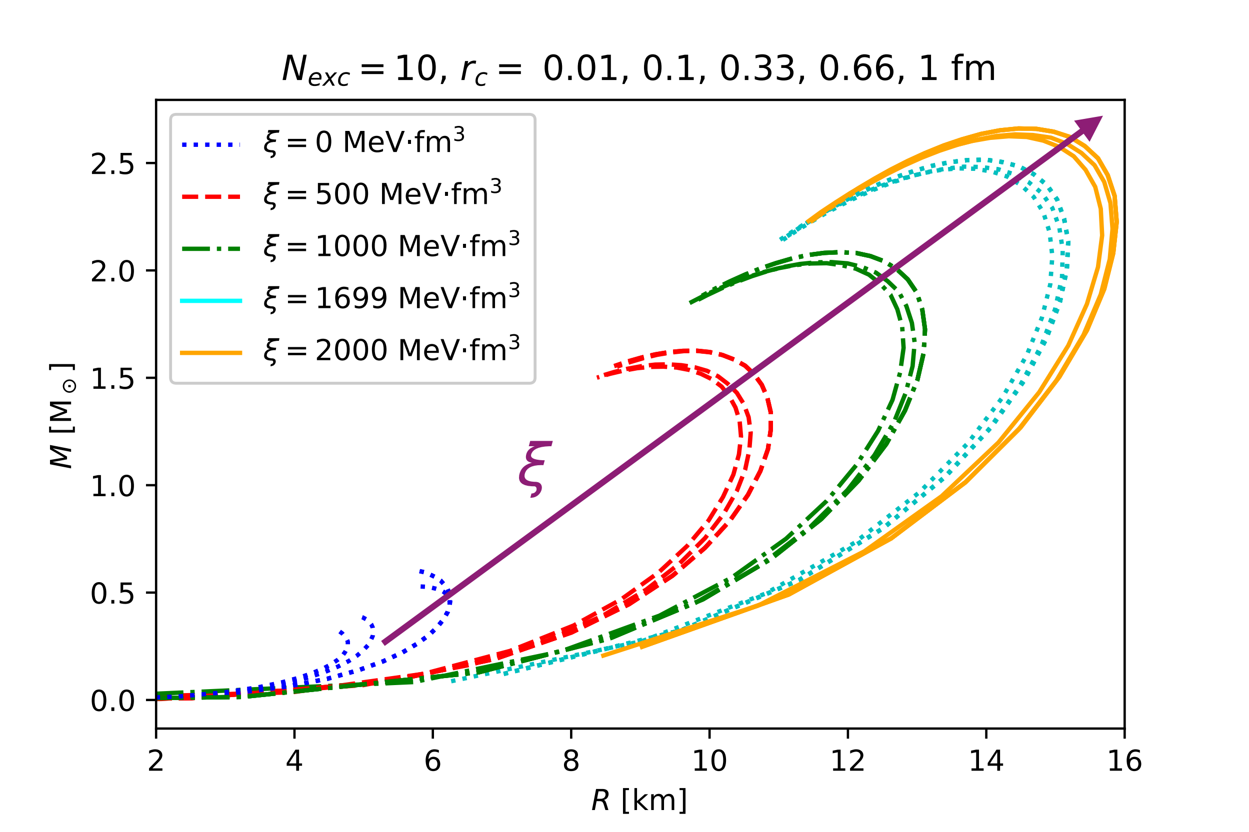
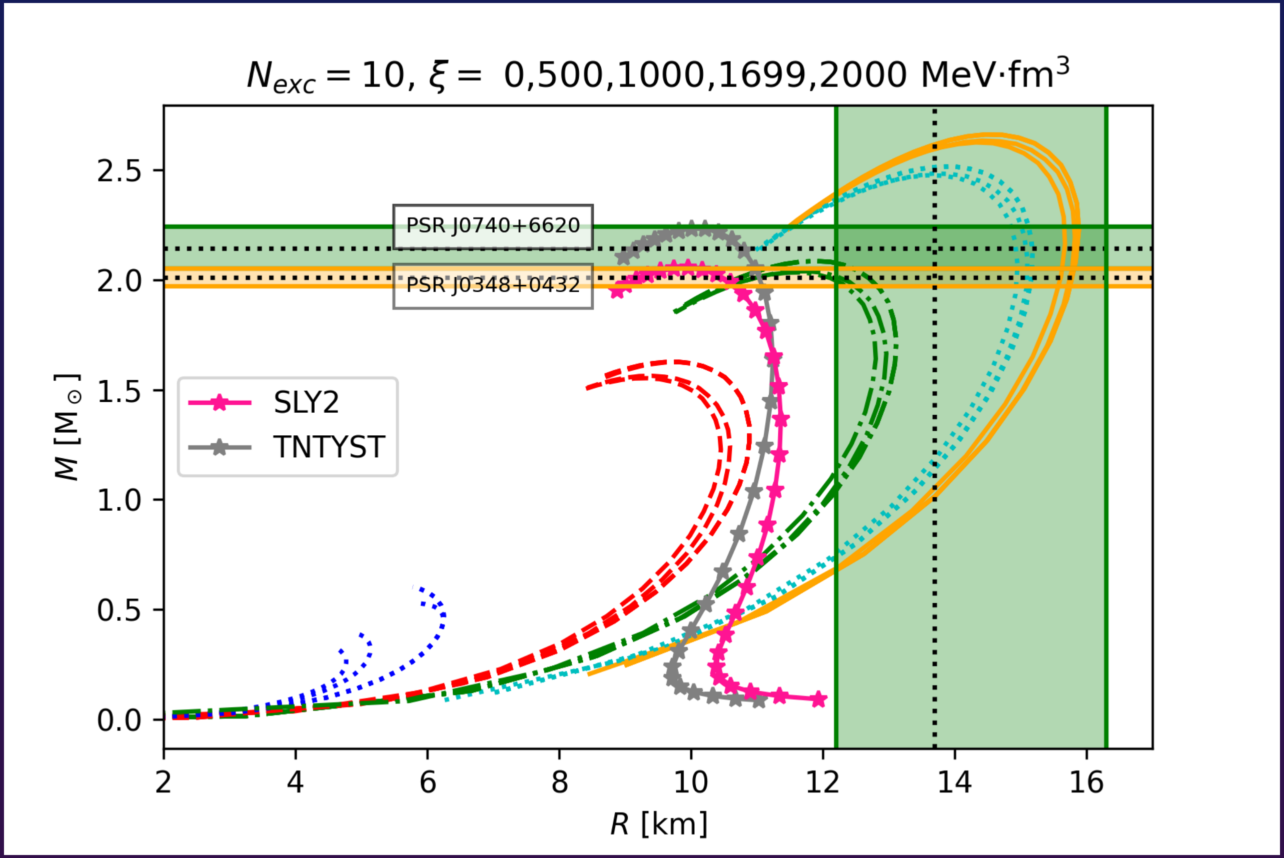


M-R diagrams

+ measurement data

+ 2 more realistic EoSs



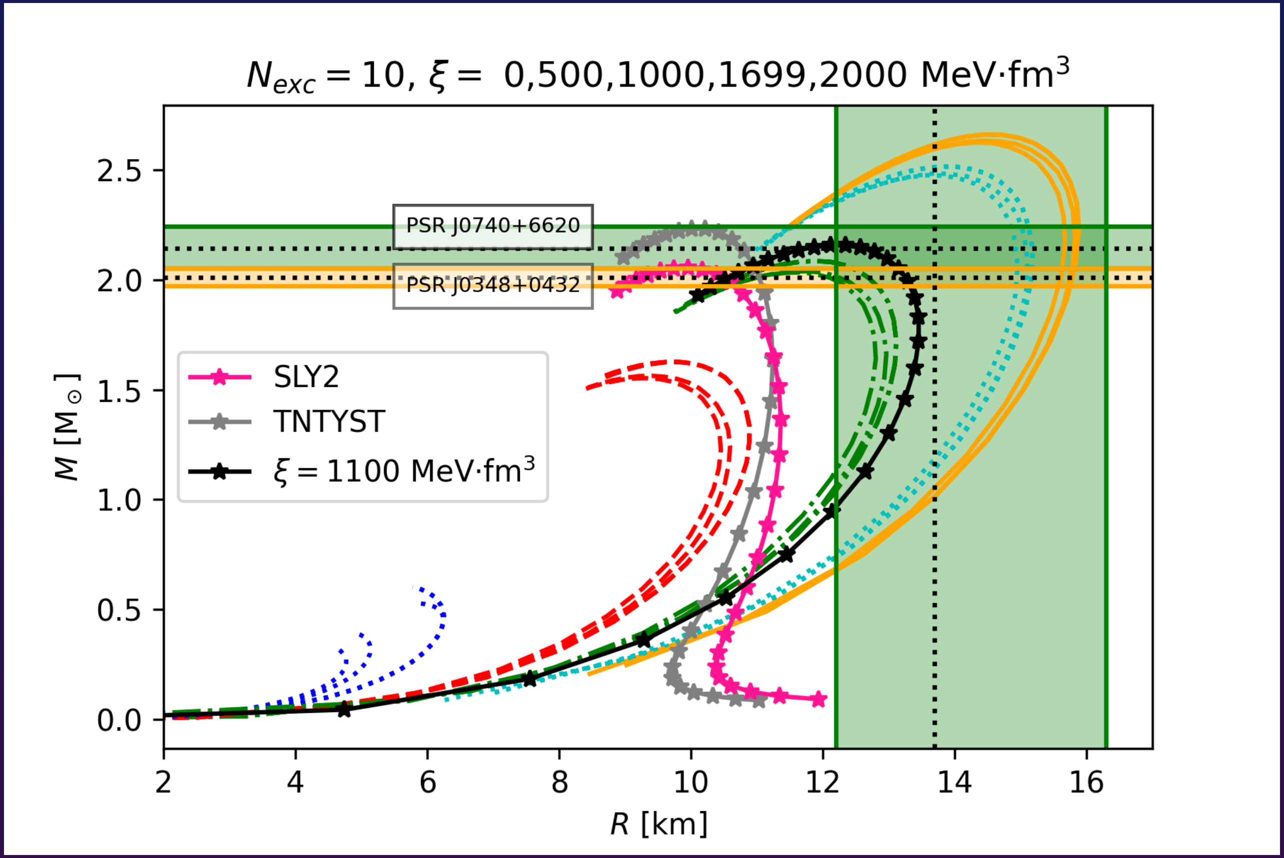
M. C. Miller et al 2021 ApJL 918 L28
H.T. Cromartie et al., Nat. Astron. 4, 72 (2019)
J. Antoniadis et al., Science 340, 1233232 (2013)

M-R diagrams

+ measurement data

+ 2 more realistic EoSs

+ approximation for ξ



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Summary

- Model with the possibility of probing **beyond standard model physics**
- One **extra** spatial **compactified dimension**

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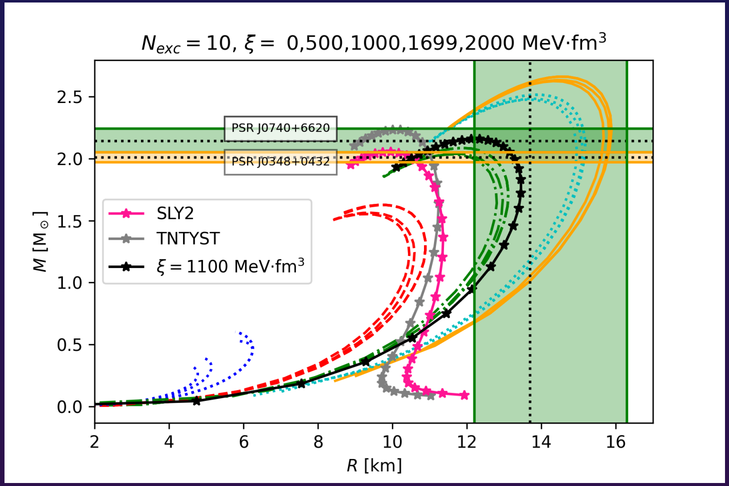
Ordinary **mass** can be described as **quantized 5thD momenta**



- Effective nuclear field theory with **linear** repulsive **potential**

- It is **possible** to **build** compact stars with **realistic** properties
- **Constraints** on the **size of** possible **extra dimensions** could be given