An Application of the Non-extensive Phenomena: The Soft+Hard Model at Various Energies

G.G. Barnaföldi, G. Bíró, T.S. Biró, P. Ván, K. Ürmössy
Heavy-Ion Research Group, Wigner Research Centre for Physics of the HAS
Address: PO Box 49, H-1525, Budapest, Hungary
E-mail: barnafoldi.gergely@wigner.mta.hu

Can we separate 'soft' & 'hard' components in HIC?

Abstract:

Hadron spectra measured in high-energy collisions present distributions which can be derived from the non-extensive statistical and thermodynamical phenomena. Based on earlier theoretical developments, it seems, the methods are very applicable for jets hadronization processes in electron-positron, proton-proton, and even in heavy-ion collisions.

Here, we present what can we learned from the recent theoretical and phenomenological developments: transverse momentum spectra and azimuthal anisotropy (v_2) of charge averaged pions and kaons stemming from high-energy collisions form RHIC to LHC energies, which are described analytically in a 'soft+hard' model. In this model, we propose that hadron yields produced in heavy-ion collisions are simply the sum of yields stemming from jets (hard yields) in addition to the yields originating from the bulk or from the Quark-Gluon Plasma (soft yields). The hadron spectra in both types of yields are approximated by the Tsallis–Pareto like distribution, and parameters q & T provided information on the system's 'soft' and 'hard' components. See more in Ref. [7].

Nature provides several power-law-tailed distributions especially for high-energy physics. In a specific matter, in the quark-gluon plasma phase – created in high-energy nuclear collisions – an exponential (thermal) distribution is also observed at the low-momentum regime. Recently, due to new developments in thermodynamics [1-6] this Janus-face behavior might be understood via generalized entropy formula which connects the two above distribution in one global description.

Mathematical suggestions for entropy formulas, different from the well-known logarithmic formula due to Boltzmann–Gibbs statistics are numerous. Two most cited stems from Alfred Rényi (1961) and Constantino Tsallis (1988). This cut-power-law formula was also suggested by Rolf Hagedorn as an interpolating function between Boltzmann–Gibbs and power law distributions. Tsallis–Pareto function contains a parameter denoted by *q* beside the temperature-like parameter, *T*. We elaborated a new chain of thoughts connecting the *q*-entropy formula with fundamental thermodynamical principles and derived that $q=1-1/C+\Delta T^2/T^2$ is related to the heat capacity of the heat reservoir. This correspondence is exact for ideal systems, and can be viewed as an approximation for the general case. The Universal Thermostat Independence (UTI) principle presented and used in Refs. [1,2,5,6] guides in the construction of the optimal finite-size corrected entropy formula characterized by the heat capacity, C of the reservoir and fluctuation of the temperature, ΔT.

Why do we need Tsallis–Pareto?

Hadron spectra measured in high-energy nuclear collisions obey Boltzmann–Gibbs statistics at low transverse momenta, p_T, while at high momenta the statistical distributions approach the power law. Tsallis–Pareto distribution is a kind which follows both in the low and high momentum limits respectively. Especially, if the system size getting smaller (Fig. 1).

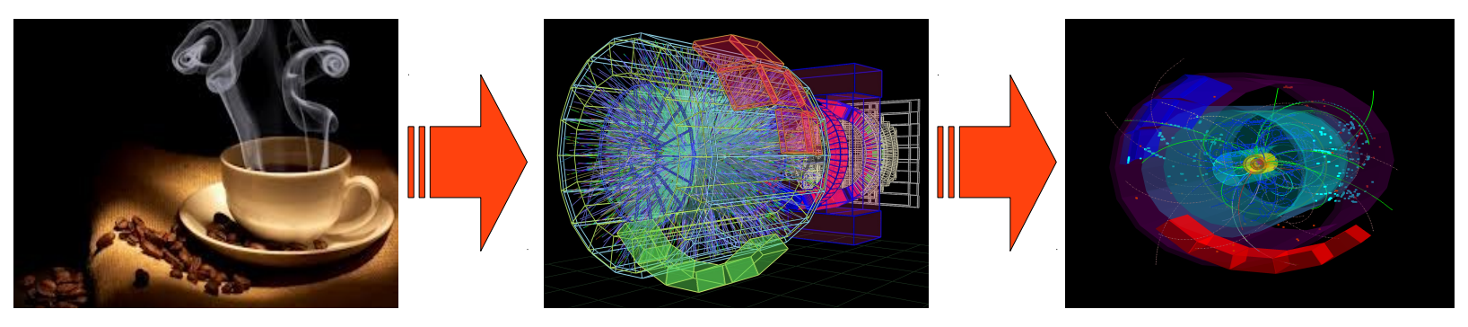


Fig 1: Thermodynamical systems from Avogadro number to 10-100 d.o.f.

Within the framework of the non-extensive statistical approach, this behavior can be explained, in addition parameters *q* and *T* carry physical information on system's size & fluctuations (Fig. 1,2).

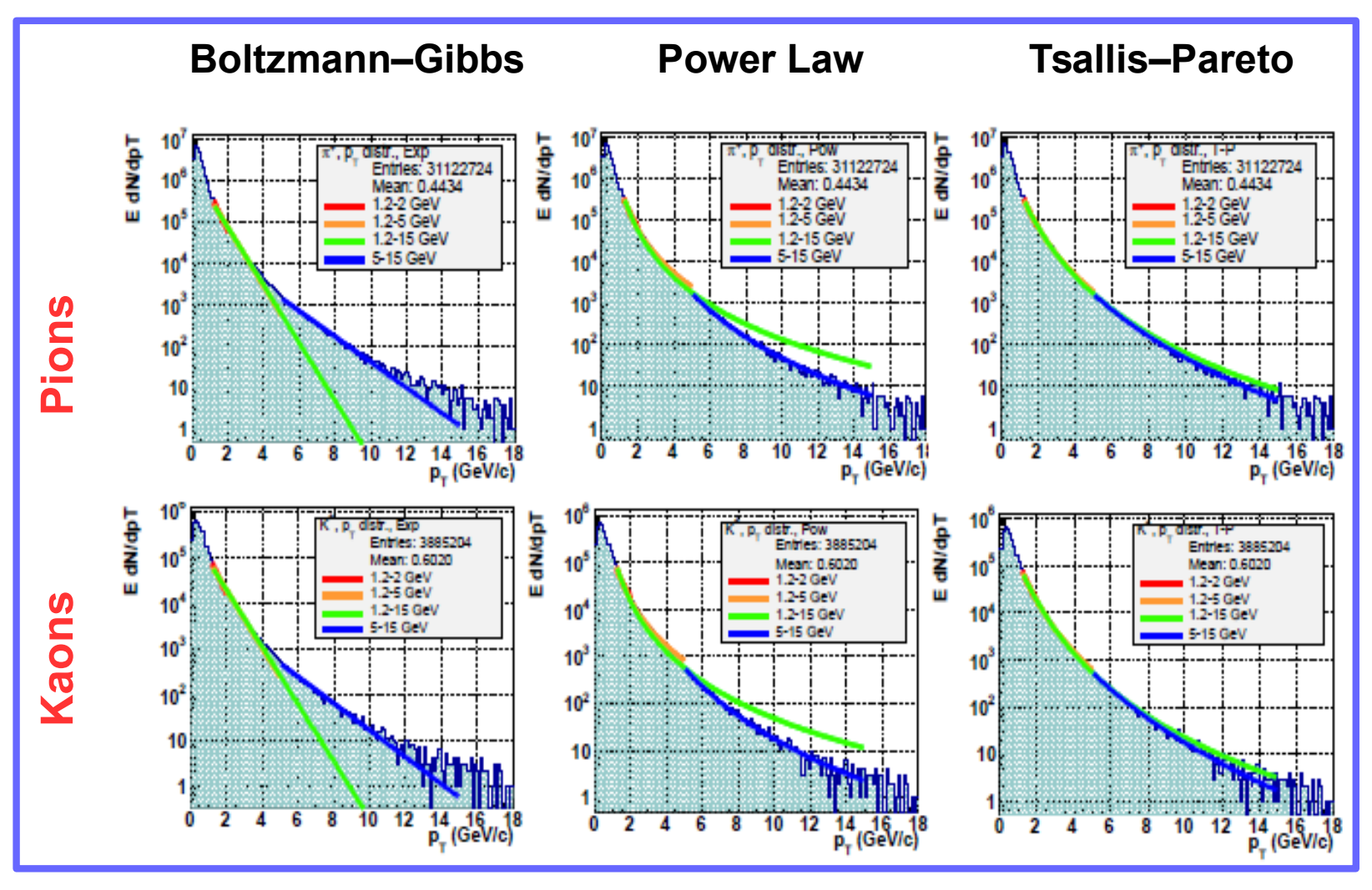
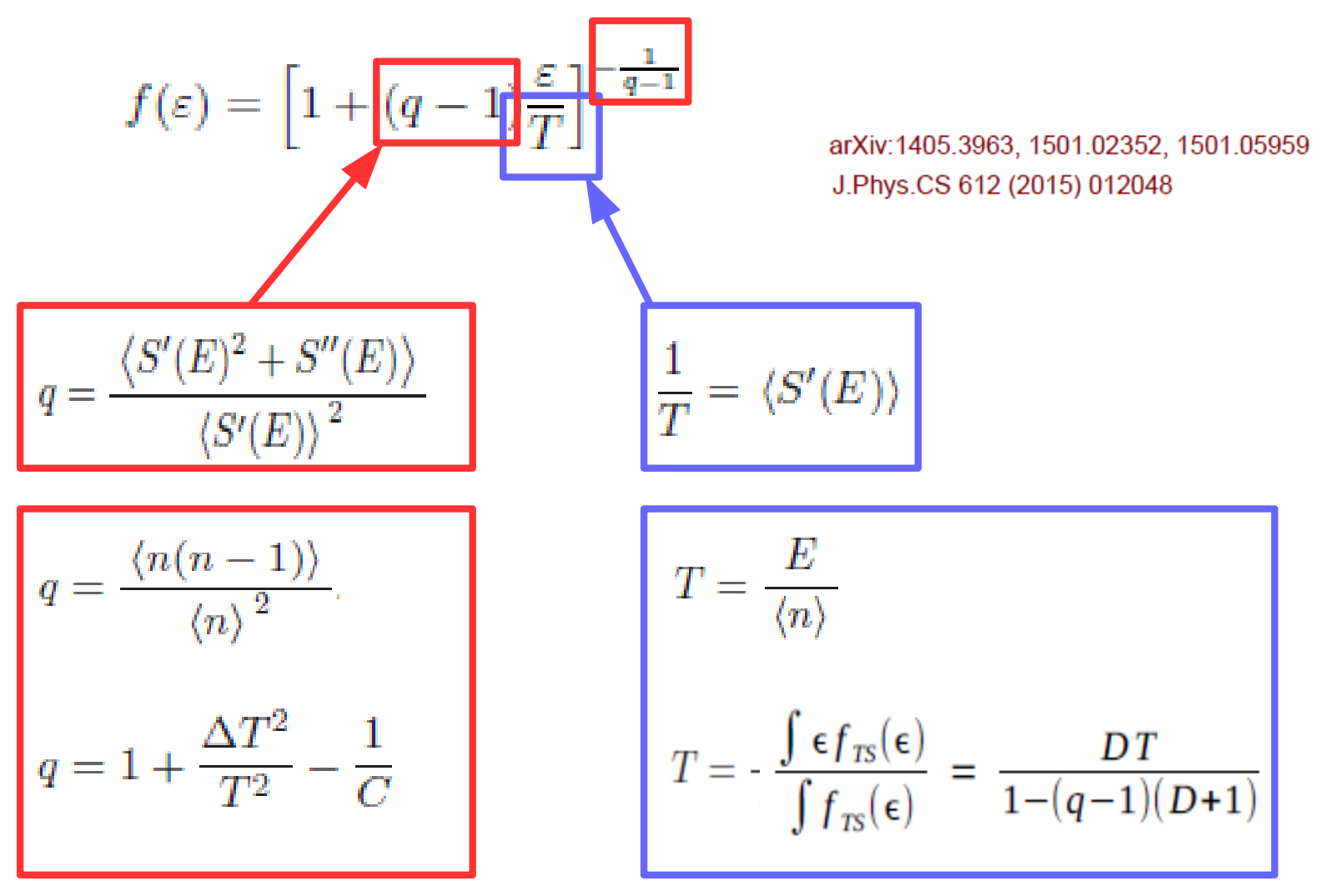


Fig 2: PYTHIA8-generated identified pion and kaon spectra, fitted by Boltzmann–Gibbs, Power law and Tsallis–Pareto distributions.

	[1,2:2] GeV/c	[1,2:5] GeV/c	[1,2:15] GeV/c	[5:15] GeV/c
Exp	112,37/29,81/27,34	623,89/130,48/109,26	254,12/61,71/48,13	3,01/1,44/1,45
Pow	1,71/0,98/0,47	161,27/55,68/56,08	214,12/76,92/77,26	1,37/1,144/0,91
TP	0,45/1,19/0,56	12,21/5,55/11,06	10,39/4,37/7,77	1,14/0,97/0,91

Table 1: The χ^2 values of the fitted pion/kaon/proton spectra for Boltzmann–Gibbs (Exp), Power law (Pow) and Tsallis – Pareto (TP) cases.

The statistical/thermodynamical definition of the *q* and *T* parameters of the Tsallis – Pareto distribution are given below, as explained in Refs. [1-6]:



$$f(\varepsilon) = \left[1 + \boxed{(q-1)}\frac{\varepsilon}{\boxed{T}}\right]^{\boxed{-\frac{1}{q-1}}}$$

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$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

$$\frac{1}{T} = \langle S'(E)\rangle$$

$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$

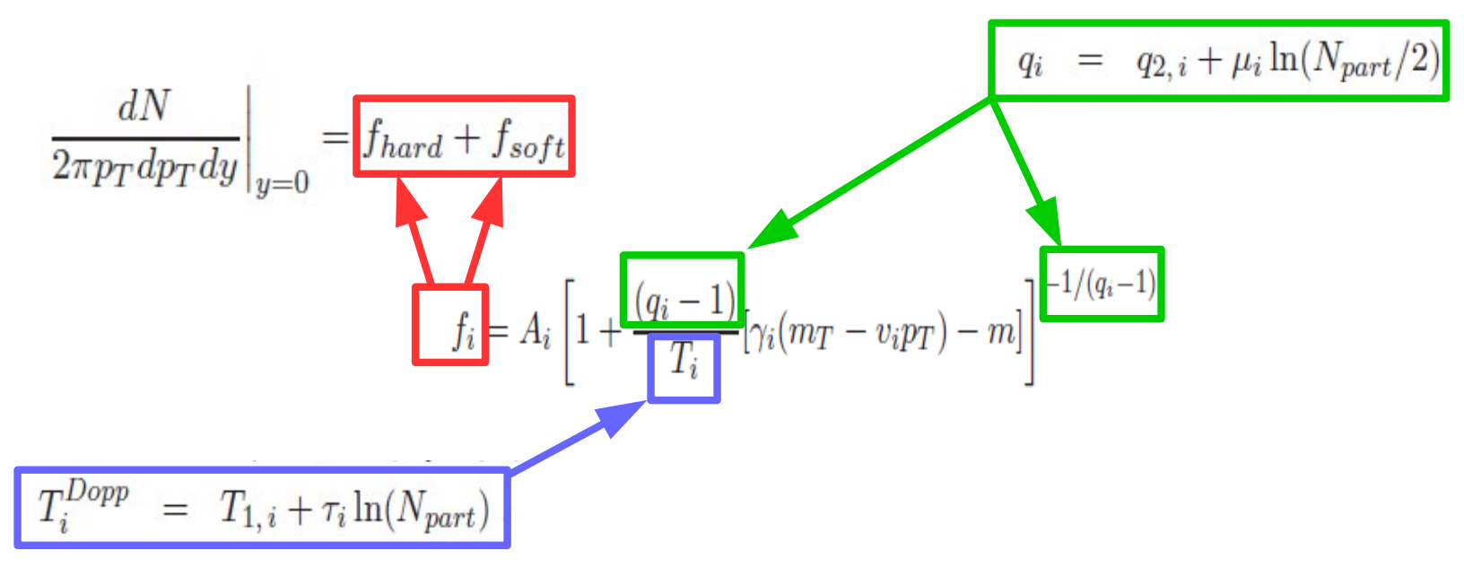
The Soft+Hard Model

Identified hadron spectra measured in electron-positron or in proton-proton collisions present clear Tsallis–Pareto distribution. However in high-energy heavy-ion collisions, it is even hard to fit measured yields by the pure or combined Boltzmann–Gibbs, power low, and Tsallis–Pareto distributions.

Since *q* & *T* parameters have physical meaning, we suggested to fit a double Tsallis–Pareto on the soft and hard components separately with the lowest χ^2 for the joint function,

$$p^0\frac{dN}{d^3\mathbf{p}} = p^0\frac{dN}{d^3\mathbf{p}}^{\text{hard}} + p^0\frac{dN}{d^3\mathbf{p}}^{\text{soft}} .$$

Fits suggested to have a minimal χ^2 at the soft/hard limit p_{T0}= 3±1 GeV/c, which was fixed, while we were obtaining the parameter values. Earlier results suggested an N_{part} scaling, which were also taken into account in the fit function as in Ref [7],



$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \boxed{f_{hard} + f_{soft}}$$

$$\boxed{f_i} = A_i\left[1 + \frac{\boxed{(q_i-1)}}{\boxed{T_i}}[\gamma_i(m_T - v_i p_T) - m]\right]^{\boxed{-1/(q_i-1)}}$$

$$q_i = q_{2,i} + \mu_i \ln(N_{part}/2)$$

$$T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})$$

where we used usual notations for the parameters

$$T_i^{Dopp} = T_i\sqrt{\frac{1+v_i}{1-v_i}} \quad \& \quad m_T = \sqrt{p_T^2 + m^2} \quad \& \quad \gamma_i = 1/\sqrt{1-v_i^2} .$$

Based on the analysis of the ALICE, CMS, and PHENIX data [7] the N_{part}-scalings of parameters *q* & *T* are plotted on Fig. 3. Values summarized in Table 2.

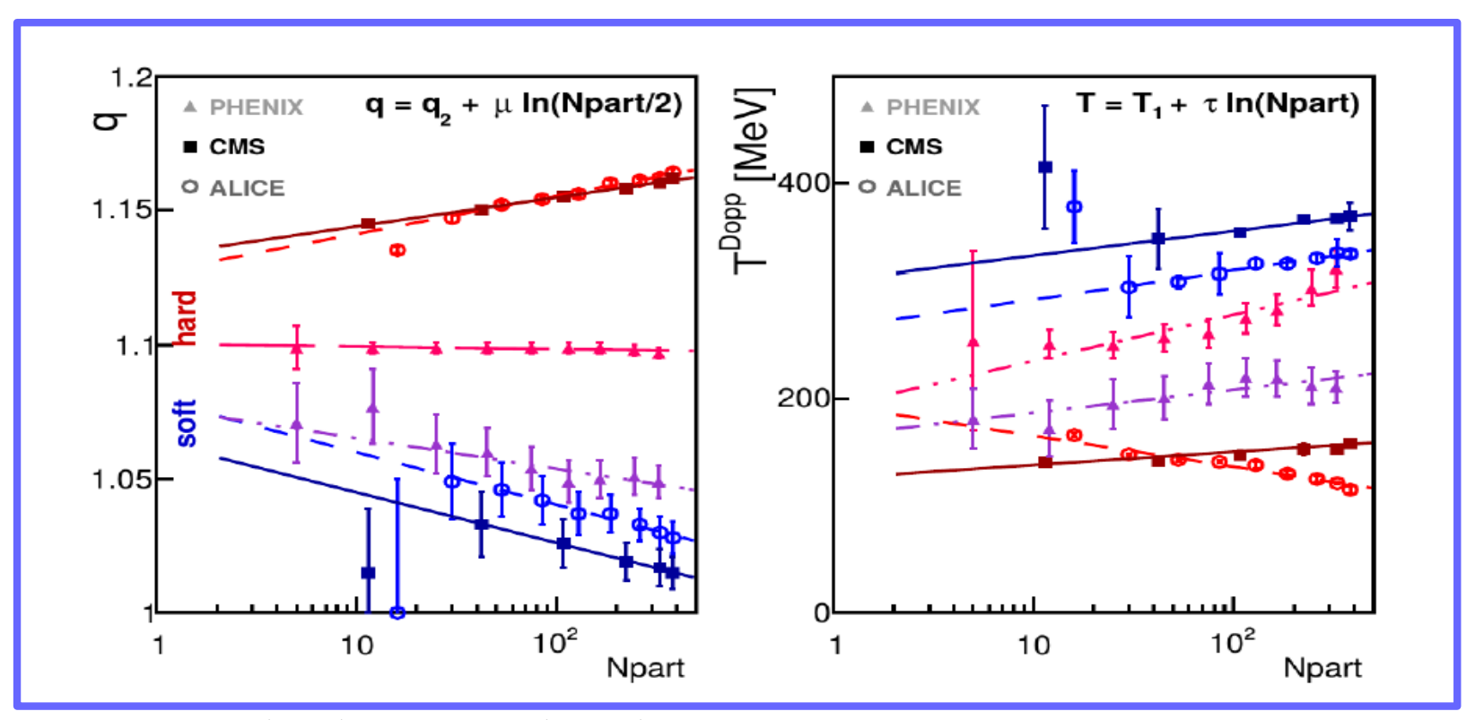


Fig 3: Hard (red) and soft (blue) double Tsallis–Pareto parameter values for pion yields measured by ALICE (dashed), CMS (solid), and PHENIX (dash-dotted), including N_{part} scaling. See Ref [7] for data and details.

	$q_{2,soft}$	$q_{2,hard}$	μ_{soft}	μ_{hard}
CMS	1.058 ± 0.025	1.136 ± 0.001	-0.008 ± 0.005	0.005 ± 0.0003
ALICE	1.074 ± 0.018	1.131 ± 0.002	-0.009 ± 0.004	0.006 ± 0.0006
PHENIX	1.073 ± 0.016	1.100 ± 0.002	-0.005 ± 0.004	0.000 ± 0.0006

	T_1^{soft} [MeV]	T_1^{hard} [MeV]	τ_{soft} [MeV]	τ_{hard} [MeV]
CMS	310 ± 20	126 ± 5	9.9 ± 3.7	5.3 ± 0.8
ALICE	266 ± 16	194 ± 2	11.5 ± 2.9	-12.5 ± 0.5
PHENIX	165 ± 26	192 ± 20	9.3 ± 5.5	18.7 ± 4.6

Table 2: Double Tsallis–Pareto parameter values for pion yields measured by ALICE, CMS, and PHENIX data [7], including N_{part} scaling.

Results on Spectra and Flow

Tsallis–Pareto *q* & *T* parameter values are plotted as a function of c.m. energy on Fig. 4 for central (left) and peripheral (right) PbPb and AuAu collisions at LHC and RHIC energies respectively.

Parameter *q* shows *q* → 1, for the soft components both for central and peripheral cases from RHIC to LHC energies. However in central collisions it is slightly closer to 1, than in peripheral case. This latter fact supports the presence of a thermalized hot and dense medium characterized by Boltzmann–Gibbs distribution in central AA collisions.

For central and peripheral cases hard components show a similar *q*-trend: increasing with c.m. energy and the slopes are the same. This suggests, that hard component is related to the hard interactions and therefore it is independent form the soft and/or bulk processes. On the other hand, the higher the energy the non-extensivity getting stronger, thus fluctuations and size effects generates *q*>1 values.

Parameter *T* seems almost constant values with about T_{hard}~100-150 MeV and T_{soft}~200-400 MeV.

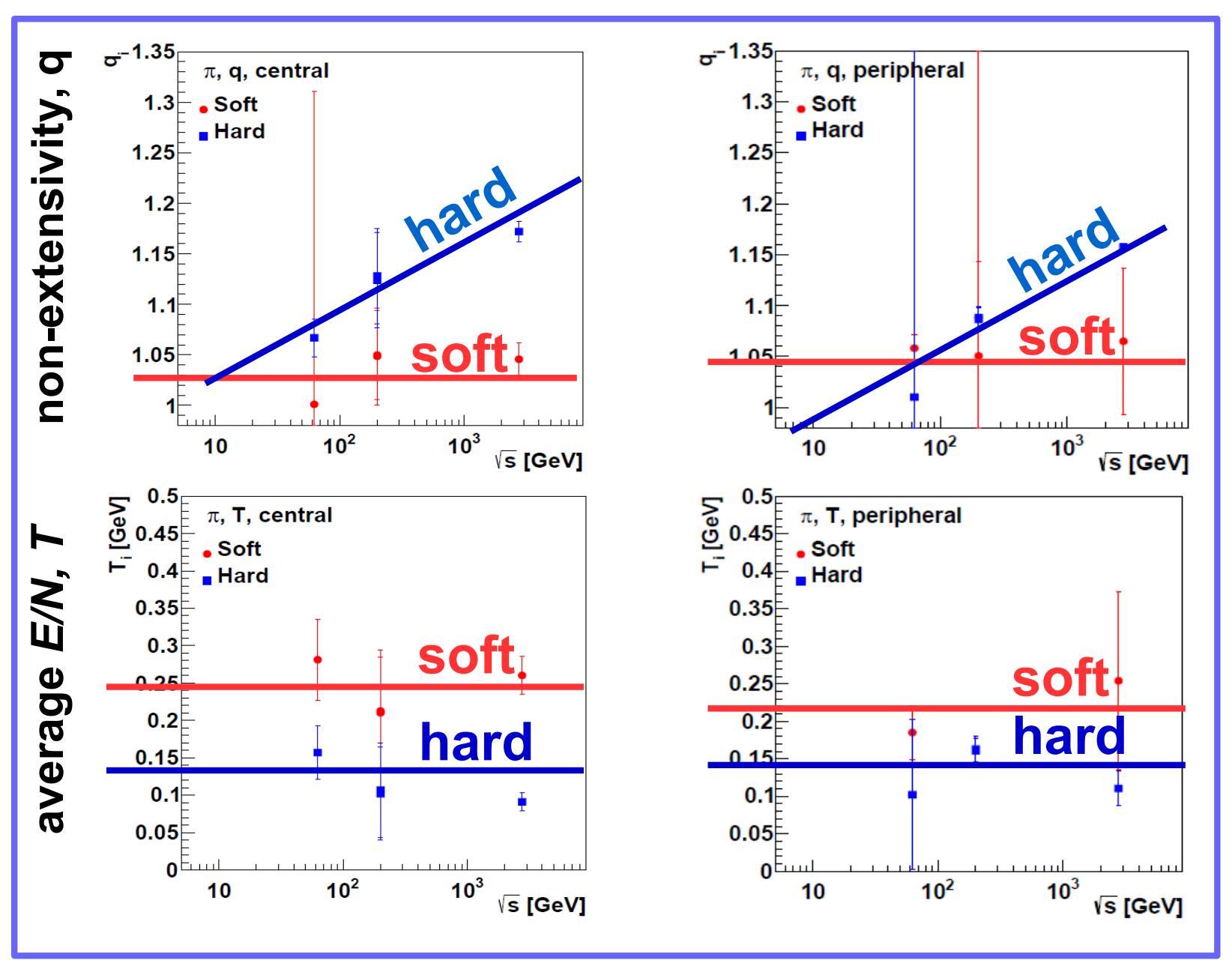


Fig 4: The c.m. energy dependence of q & T of the 'soft+hard' model in cases of central (left panel) and peripheral (right panel) AA collisions.

Using these, v_2 is also fitted. Applying its definition, comparison with experimental data were agreed well.

$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}} \qquad w_i = \frac{\delta v_i \gamma_i^3}{2T_i}\frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i(m_T - v_i p_T) - m]}$$

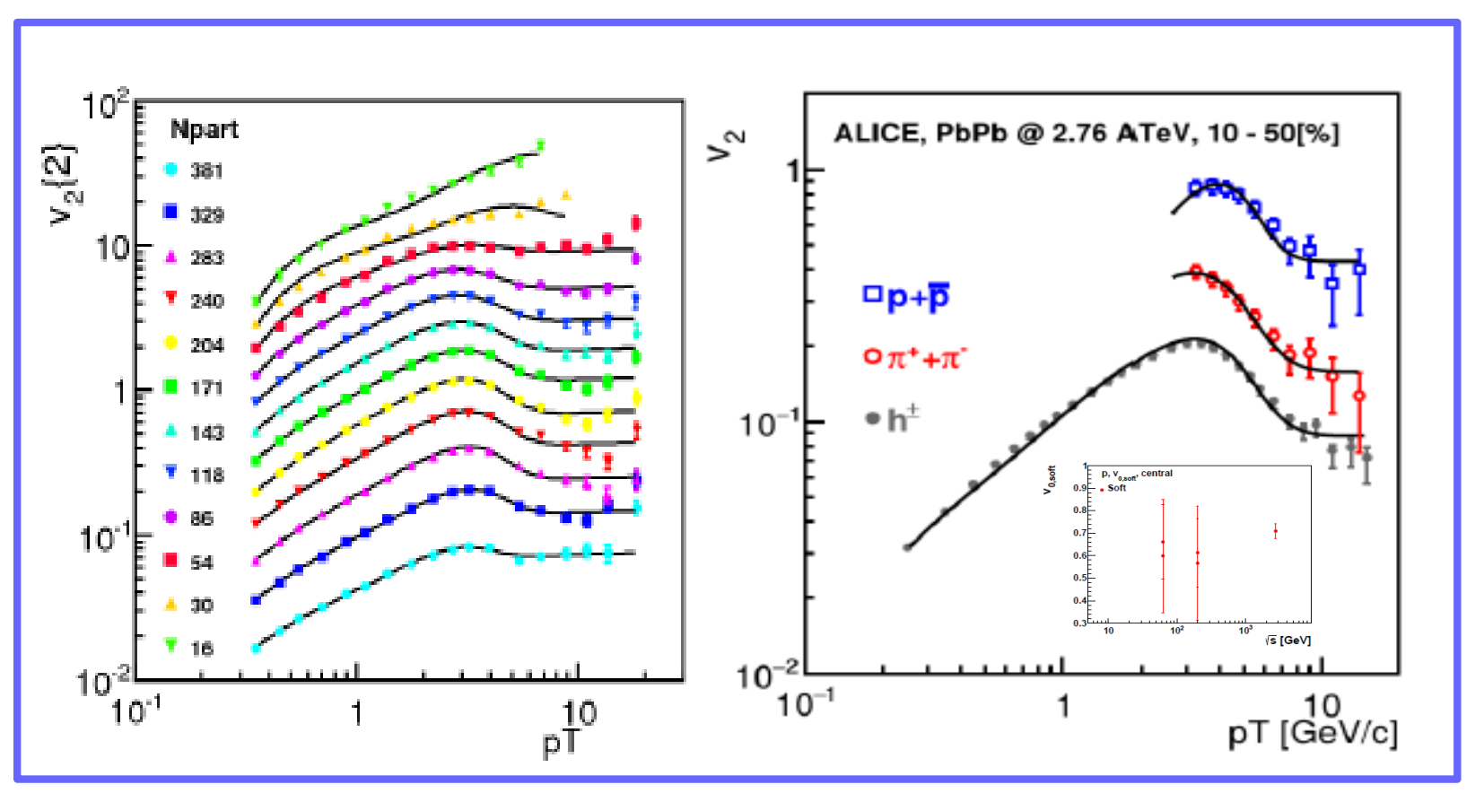


Fig 5: Comparing v_2 to experimental data: N_{part} scaling in several bins (left panel), pion, proton, and unidentified hadron v_2 values (right panel).

References

[1] **T.S.Biró, G.G.Barnaföldi, P.Ván:** *Quark Gluon Plasma connected to finite heat bath*, **EPJ A 49: 110, 2013**
[2] **T.S.Biró:** *Ideal gas provides q-entropy*, **Physica A 392: 3132, 2013**
[3] **T.S.Biró, Z.Szendi, Z.Schram:** *Quarks, Flow and Temperature in Spectra,* **J. Phys. CS 509: 012027**
[4] **K.Ürmössy, G.G.Barnaföldi, T.S.Biró:** *Microcanonical Jet-Fragmentation in Proton-Proton Collisions at LHC,* **Phys.Lett. B 718: 125, 2012**
[5] **T.S. Biró, G.G. Barnaföldi, P. Ván, K. Ürmössy:** *Statistical Power-Law Spectra due to Reservoir Fluctuations,* **arXiv:1404.1256**
[6] **T.S. Biró, G.G. Barnaföldi, P. Ván:** *New Entropy Formula with Fluctuating Reservoir,* **arXiv:1405.3813**
[7] **G.G. Barnaföldi, K. Ürmössy, G. Bíró:** *A 'soft+hard' model for Pion, Kaon, and Proton Spectra and v_2 measured in PbPb Collisions at √s = 2.76 ATeV* **J.Phys.Conf.Ser. 612 (2015) 1, 012048, arXiv:1509.06195, 1501.05959, 1501.0235, 1405.3963**

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