

Scaling properties of jets in high-energy pp collisions

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University

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Outline

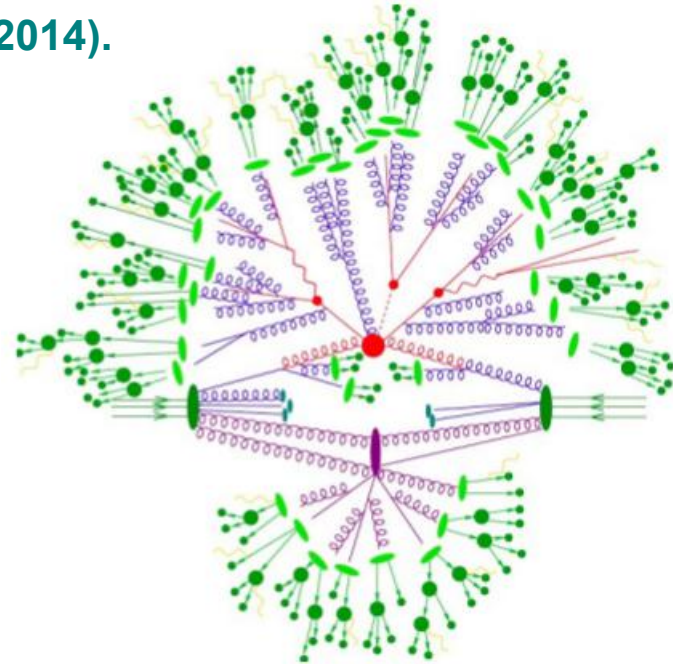
- Scaling of jet-momentum profiles with multiplicity
 - [arXiv:2008.08500](#)
- KNO-like scaling within a jet in pp collisions
 - [arXiv:2012.01132](#)

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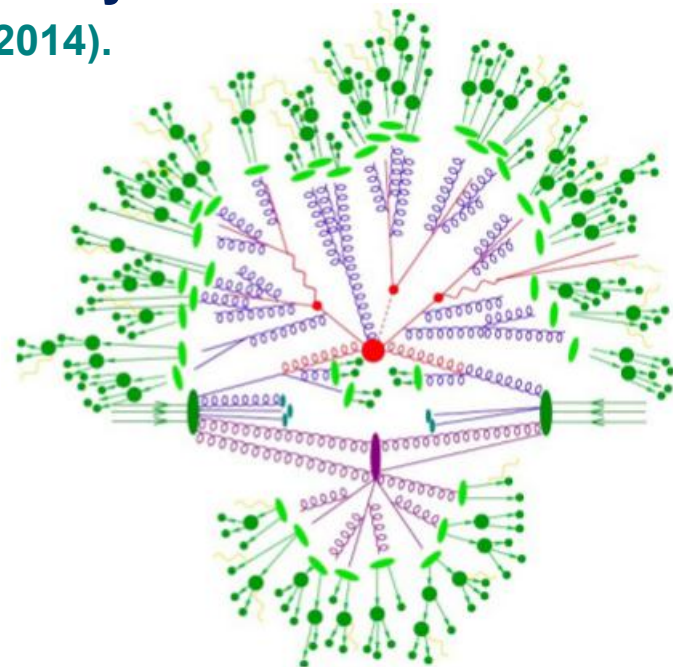
Motivation

- **Collectivity in small systems with high-multiplicity at LHC**
 - Substantial v_n , eg. [Yan-Ollitrault, PRL 112, 082301 \(2014\)](#).
- **Current understanding:**
 - QGP is not necessary for collectivity
 - Vacuum-QCD effects at the soft-hard boundary:
for instance **multiple-parton interactions (MPI)**
eg. [Schlichting, arXiv:1601.01177](#)
 - and **color reconnection (CR)** [model element]
eg. [Ortiz-Becédi-Bello, J.Phys.G 44 \(2017\)](#)



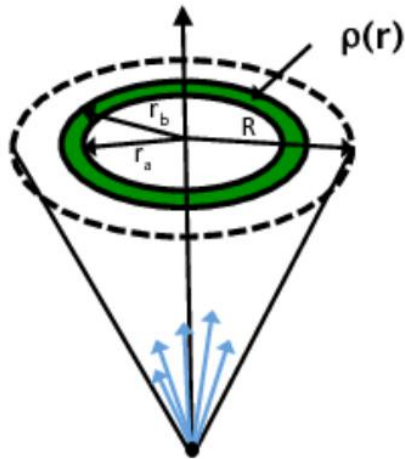
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- **Jets:**
 - **A-A**: sensitive probe of nuclear modification.
 - **pp**: No suppression expected.
However: soft and hard processes are related by MPI
=> jets can serve to study this connection



Radial jet profiles

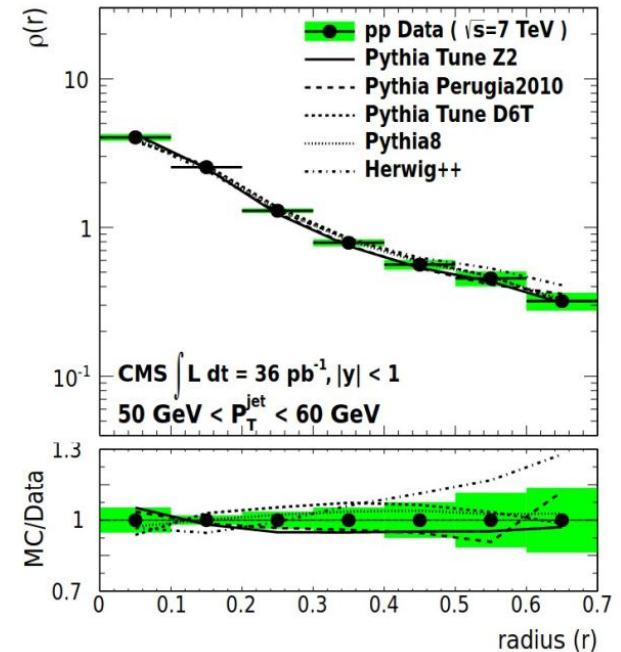
- Differential jet shape



$$\rho(r) = \frac{1}{\delta r} \frac{1}{p_T^{\text{jet}}} \sum_{r_a < r_i < r_b} p_T^i$$

$$r_i = \sqrt{(\phi_i - \phi_{\text{jet}})^2 + (\eta_i - \eta_{\text{jet}})^2}$$

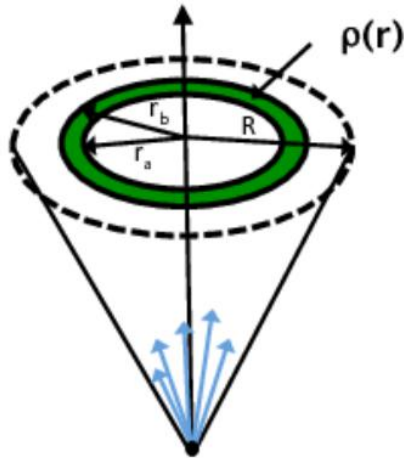
CMS, JHEP 06, 160 (2012)



- CMS@LHC pp collisions, $\sqrt{s} = 7$ TeV
- $R=0.7$ jets, $50 < p_T^{\text{jet}} < 60$ GeV/c, $|y| < 1$

Radial jet profiles

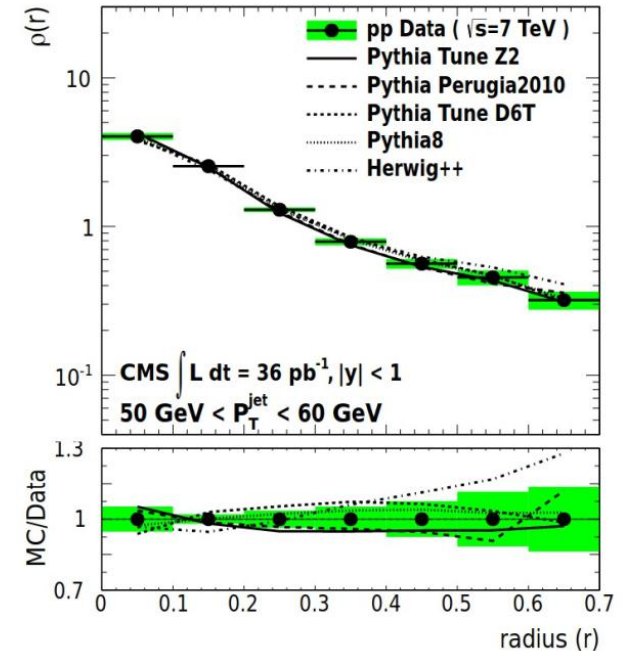
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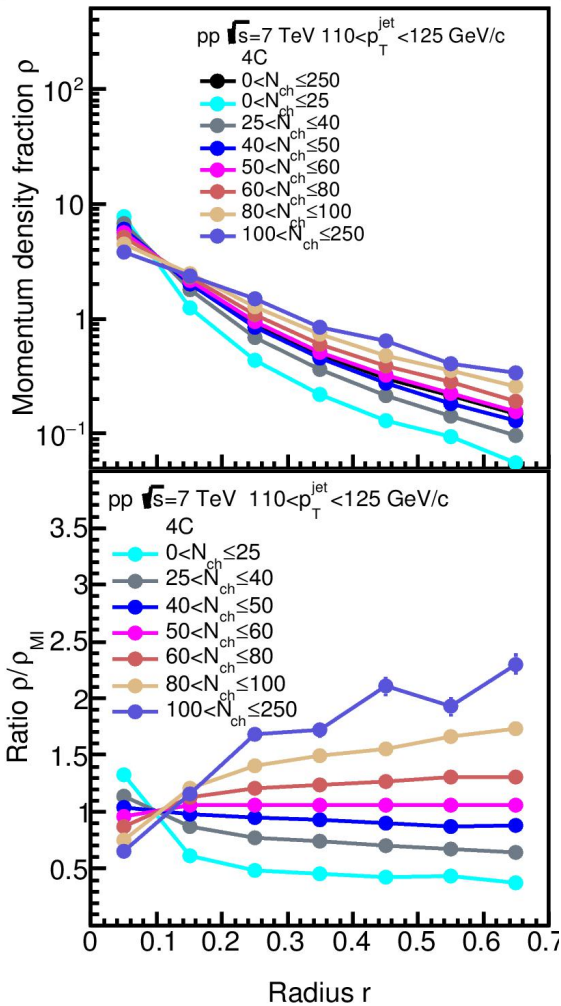
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- CMS@LHC pp collisions, $\sqrt{s} = 7$ TeV
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- Currently available LHC data are either multiplicity or transverse-momentum inclusive

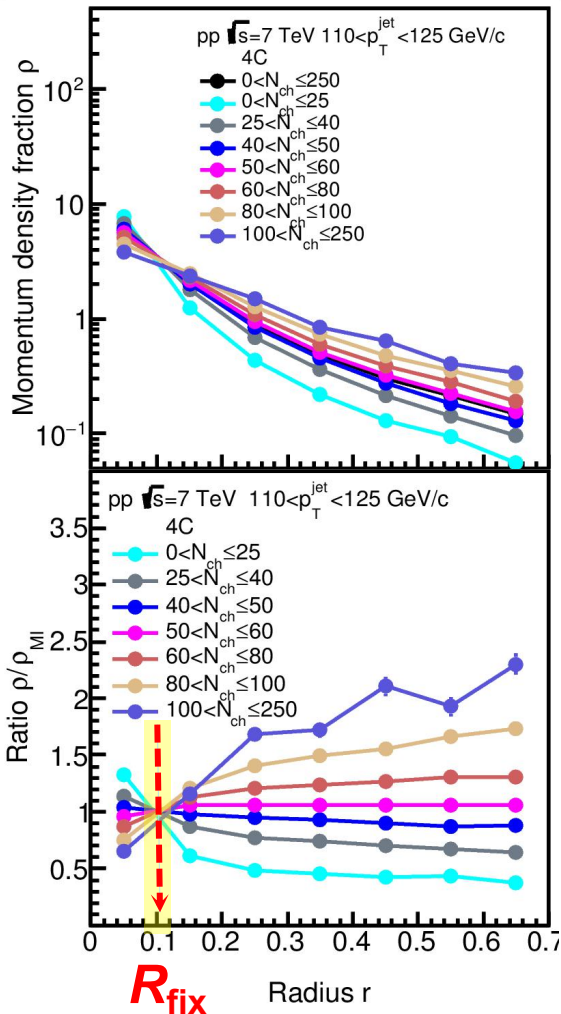
More multiplicity classes



- PYTHIA 8.2 simulations (HardQCD)
 - pp collisions at $\sqrt{s} = 7$ TeV
 - $R=0.7$ jets, $50 < p_T^{\text{jet}} < 60$ GeV/c, $|y| < 1$
- **7 multiplicity classes:**
jet profile curves intersect at a given point R_{fix}
in any given p_T^{jet} window

Z. Varga, R.V, G.G.B,
Adv.HEP 2019, 6731362 (2019)

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- **R_{fix} independent of**
 - **generator:** Pythia, Hijing++
 - **tune:** 4C, Monash, Monash*
 - **nPDF sets**
 - **CR scheme or MPI**
 - **jet algorithm:** anti- k_T , C/A, k_T

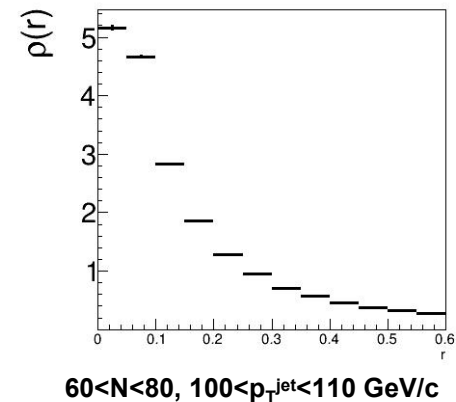
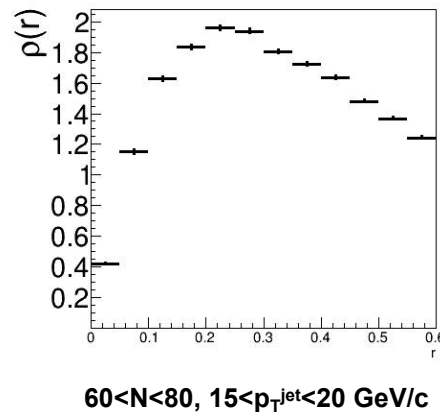
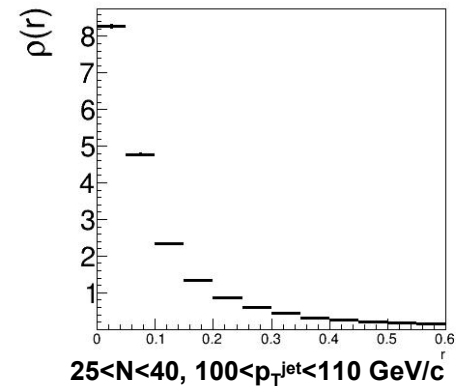
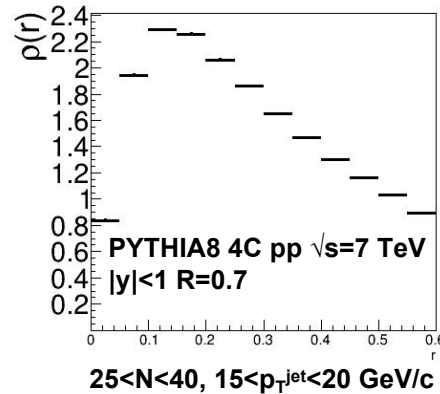
- **Is it a scaling behavior?**

Z. Varga, R.V, G.G.B,
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Parametrizing the jet profiles

- Detailed PYTHIA 8 simulations (4C)

- Jet radius: 12 bins up to $r=0.6$
- Multiplicity 6 bins up to $N=100$
- Momentum: 20 bins up to $p_{\text{T}}^{\text{jet}}=400$



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- Statistically motivated distributions:

- Poissonian distribution**

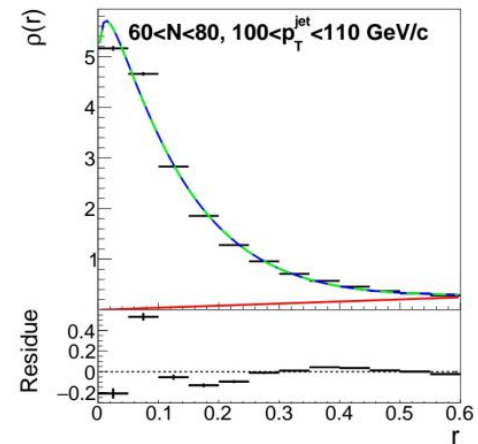
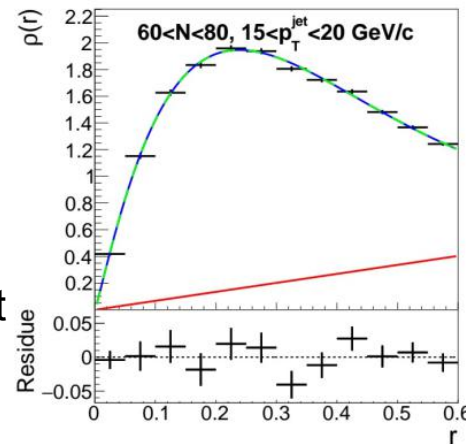
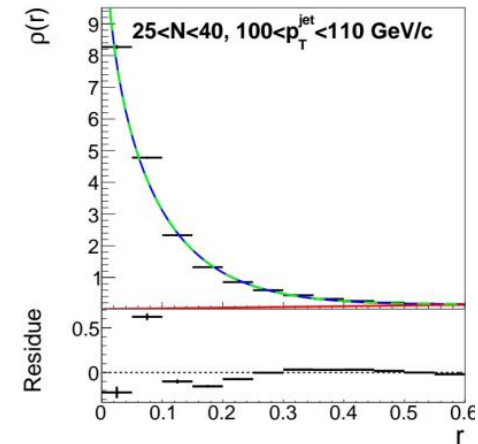
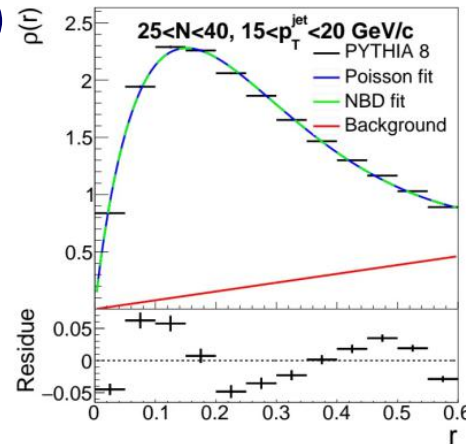
$$\rho(r) = Cr^\gamma e^{-\alpha r}$$

- NBD** (Negative binomial distribution)

$$\rho(r) = C \frac{\Gamma(rk+a)}{\Gamma(a)\Gamma(rk+1)} p^{rk} (1-p)^a$$

Note: both in the wide-jet ($p \rightarrow 1$) and narrow-jet limit ($\gamma \rightarrow -1$), NBD reduces to Poissonian

- Simultaneous fit with a $\sim br$ background



arXiv:2008.08500

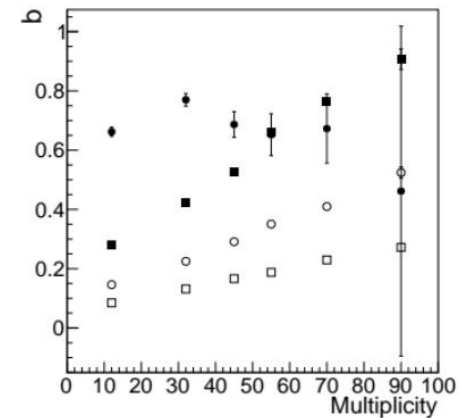
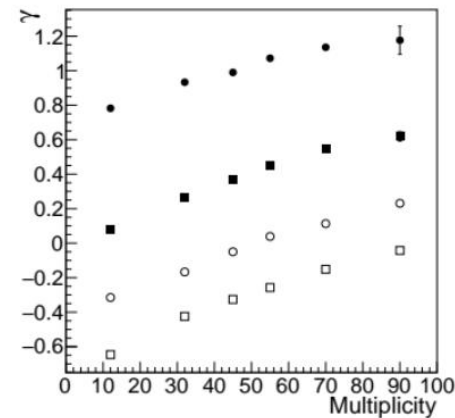
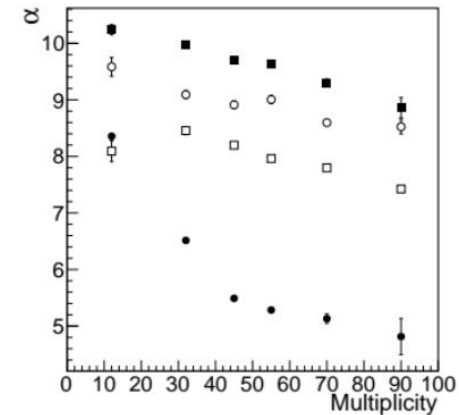
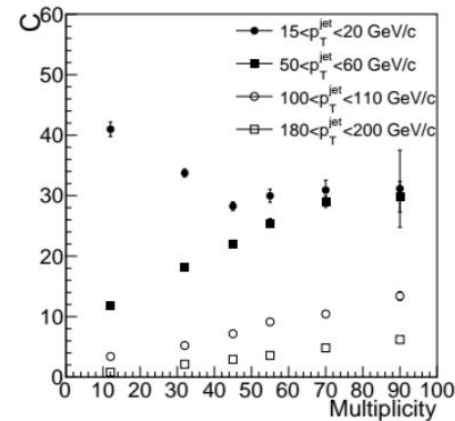
Parameters of the Poissonian fits

- Poissonian with background

$$\rho(r) = Cr^\gamma e^{-\alpha r} + br$$

- Monotonic trends observable

- Exception: lowest p_T
 - Underdetermined background fit (mostly affects b and C)
 - Leakage of jet outside $R=0.7$ (affects C)



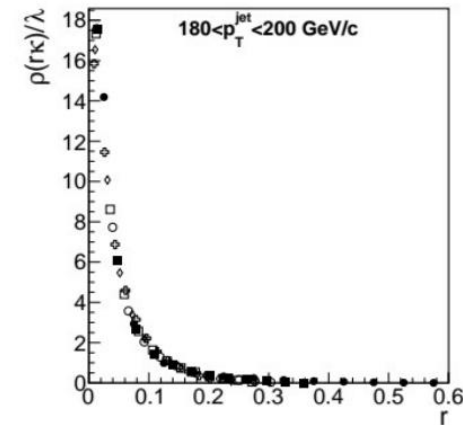
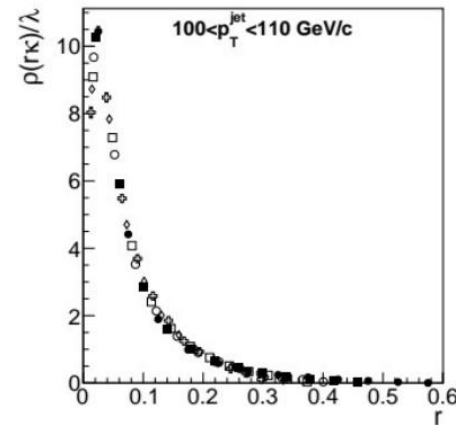
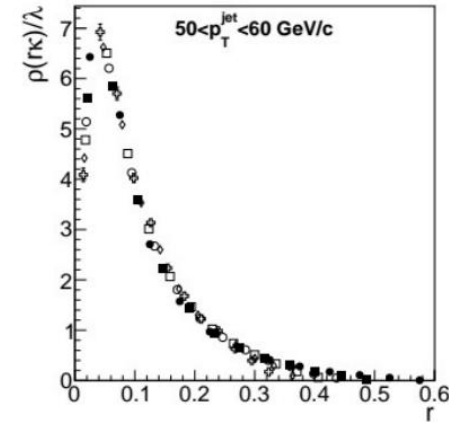
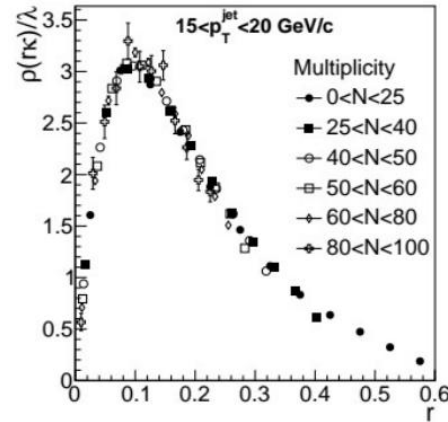
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Scaling of the jet profiles

- Scaling assumption: profiles at all multiplicities collapse into a single distribution,

$$\rho_N(r) = \lambda(N) f\left(\frac{r}{\kappa(N)}\right)$$

- Scaling is determined based on Poissonian fits
 - Chosen “good” mid-multiplicity fits, then others scaled to it minimizing χ^2



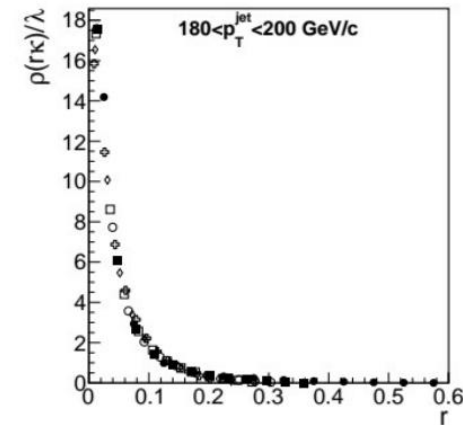
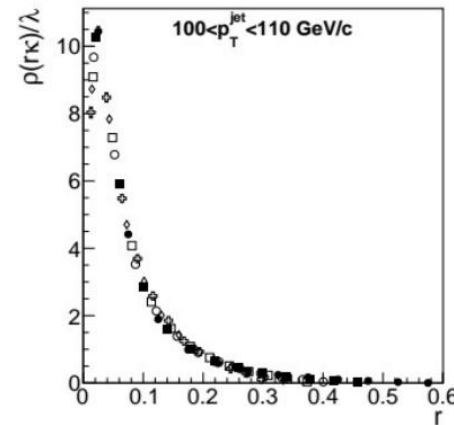
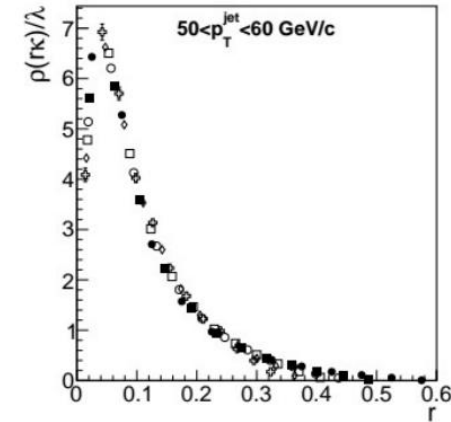
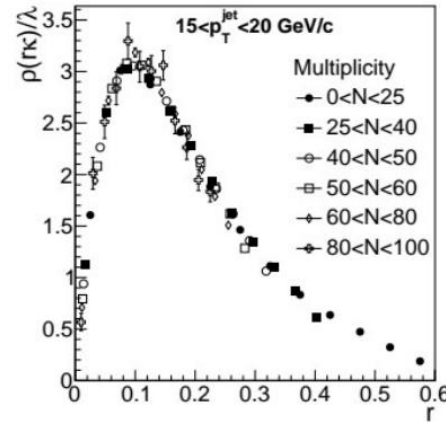
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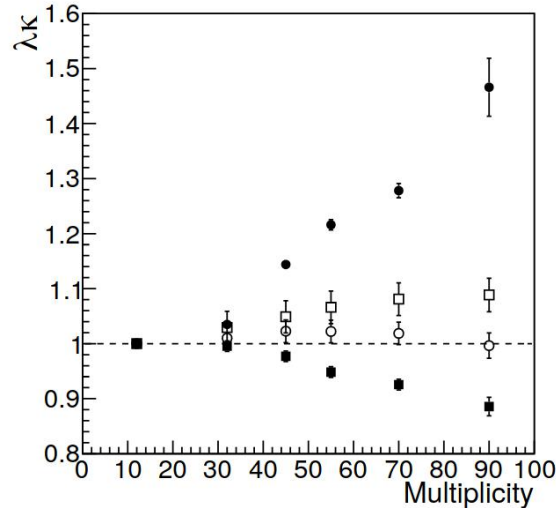
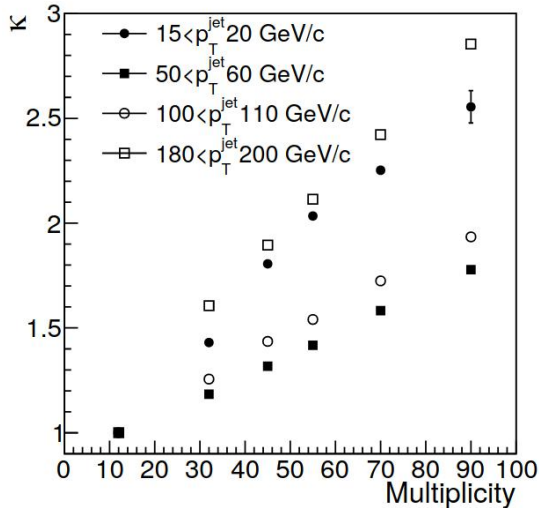
- Scaling is determined based on Poissonian fits
 - Chosen “good” mid-multiplicity fits, then others scaled to it minimizing χ^2
- The scaling works within 5-10% in the peak region**



arXiv:2008.08500

Scaling factors

arXiv:2008.08500



$$\rho_N(r) = \lambda(N) f\left(\frac{r}{\kappa(N)}\right)$$

- The scaling parameter κ is approximately linear with multiplicity
- Ideally, $\lambda\kappa \sim 1$. This is fulfilled on the 10% level except for the lowest- p_T bin
 - Low- p_T increase is because leakage increases λ
 - Slight high- p_T decrease is because background determination

How good are the Poisson fits?

- Poissonian mean:

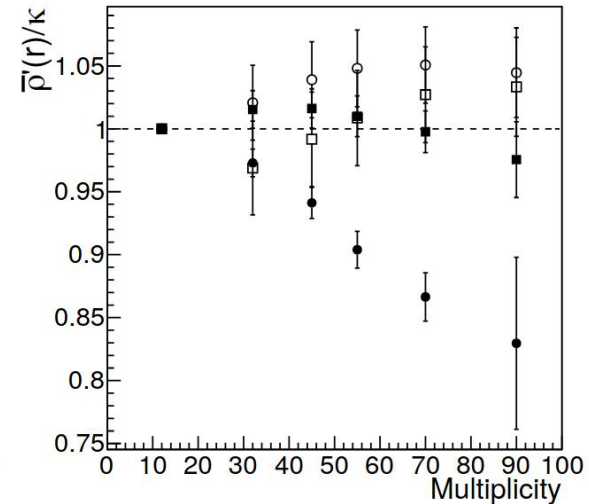
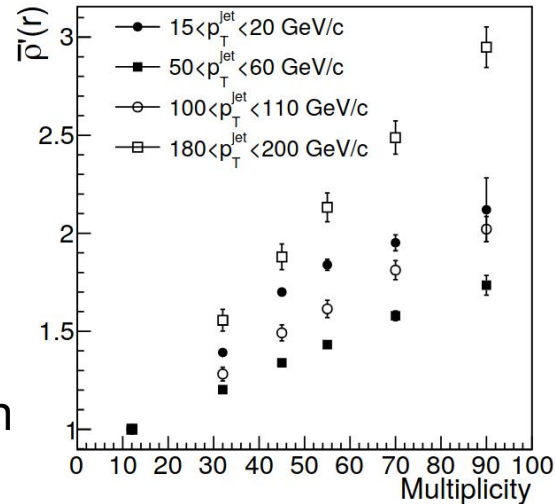
$$\bar{\rho}(r) = \frac{\gamma + 1}{\alpha}$$

- Ideally, it should scale:

$$\kappa / \bar{\rho}' \sim 1$$

where $\bar{\rho}'$ is the rescaled mean

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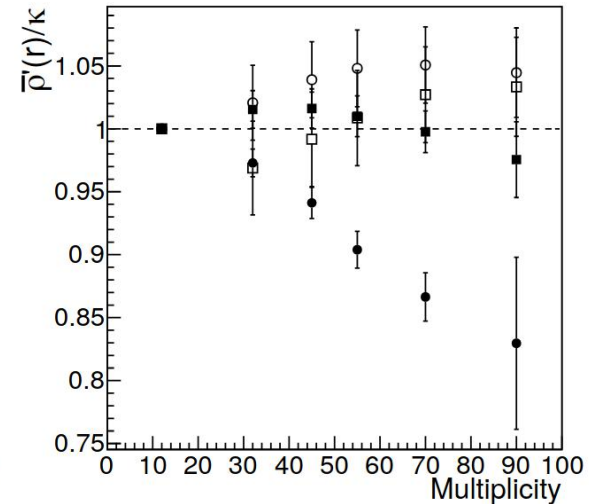
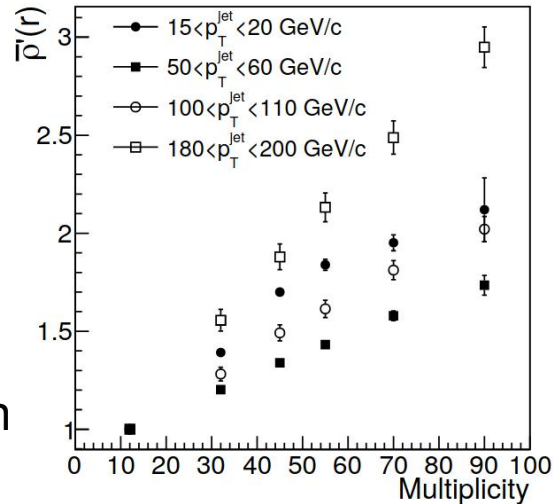
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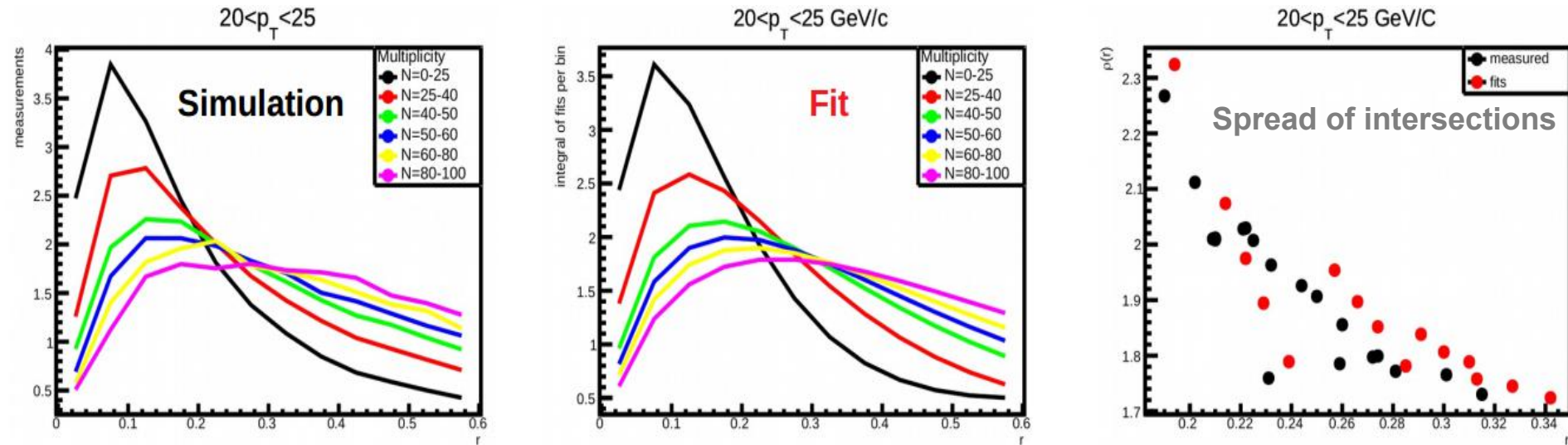
- The mean approximately scales linearly with multiplicity
- Except for the lowest p_T bin, $\kappa / \bar{\rho}' \sim 1$ within 5%
- Hence,

- » Radial profiles scale with multiplicity
- » Poissonian is an adequate description

arXiv:2008.08500



Is there really an R_{fix} ?



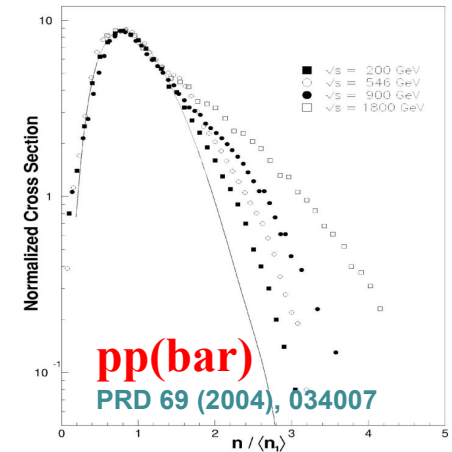
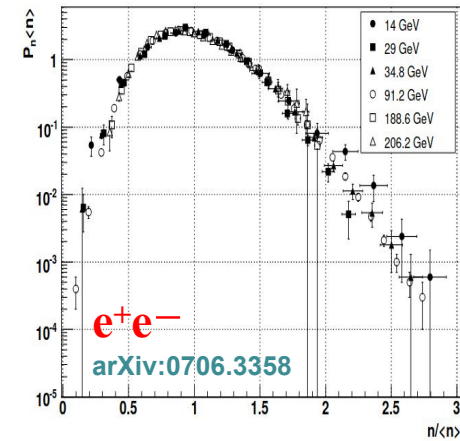
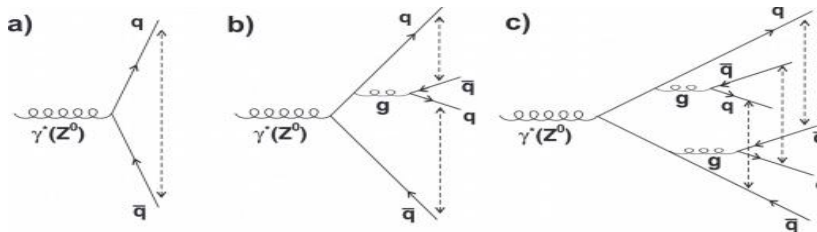
- Based on the Poisson distribution parametrization, R_{fix} is an approximate consequence of the scaling
- Note: R_{fix} would be exact if $\rho(r)$ fell linearly in the given region

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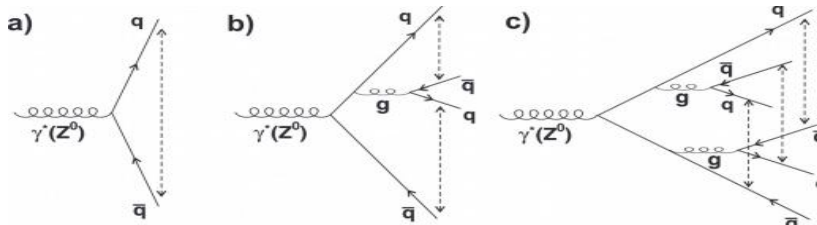
KNO-scaling and its violation

- KNO scaling: the multiplicity distribution scales with \sqrt{s}
Koba-Nielsen-Olesen, NPB 40, 317 (1972); Polyakov, Sov.Phys.JETP 32, 296 (1971)
- The KNO scaling breaks down at high \sqrt{s}
- KNO may be violated by the presence of multiple-parton interactions or overlapping color strings
Walker PRD 69, 034007 (2004); Abramovsky et al., arXiv:0706.3358

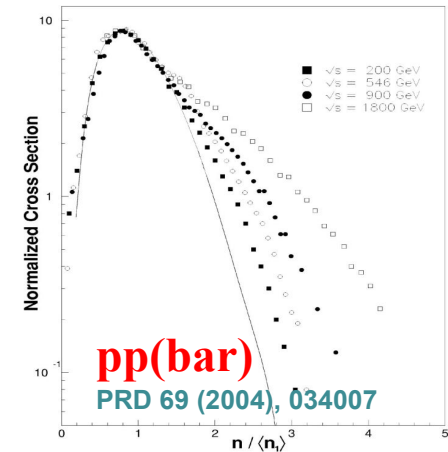
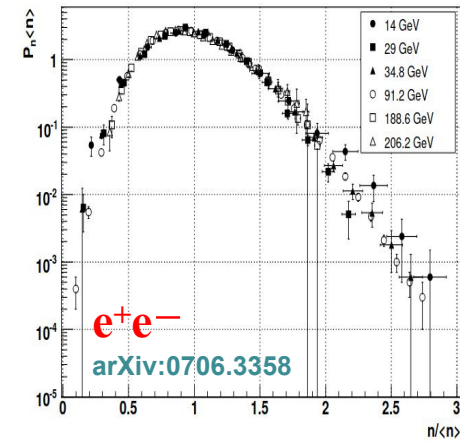


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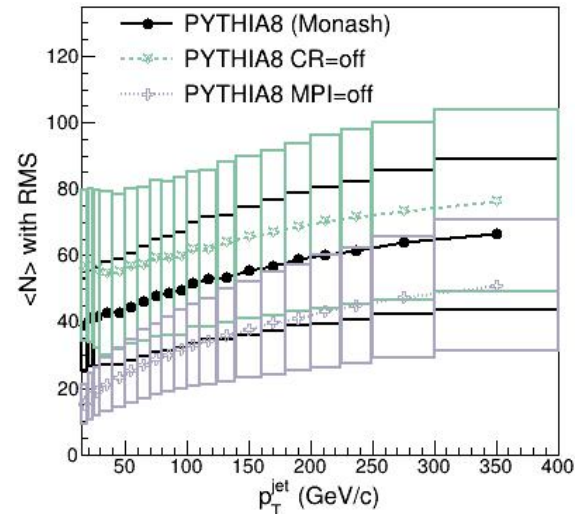
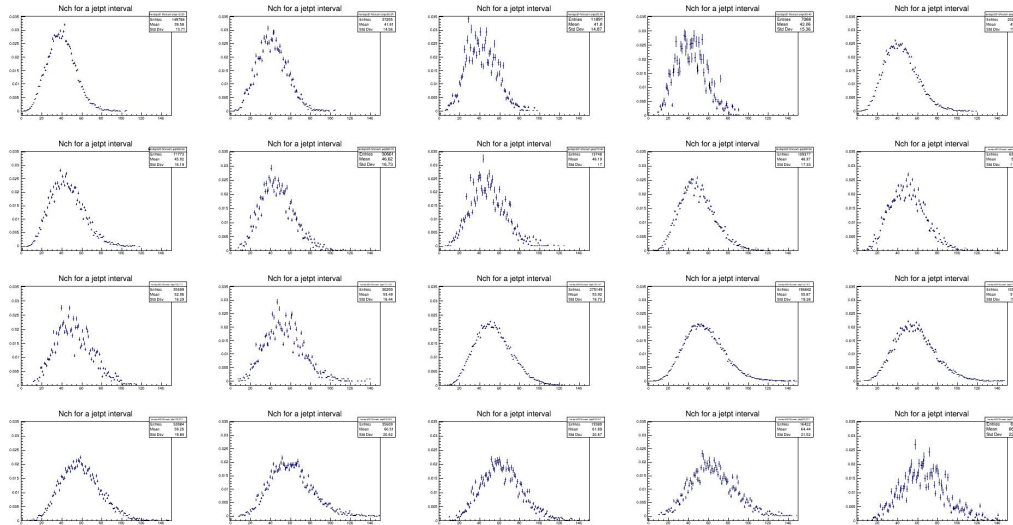
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- **Is KNO-scaling valid within a single jet?**
- **How is affected by MPI and CR?**
- **Is there a connection of KNO to radial scaling?**

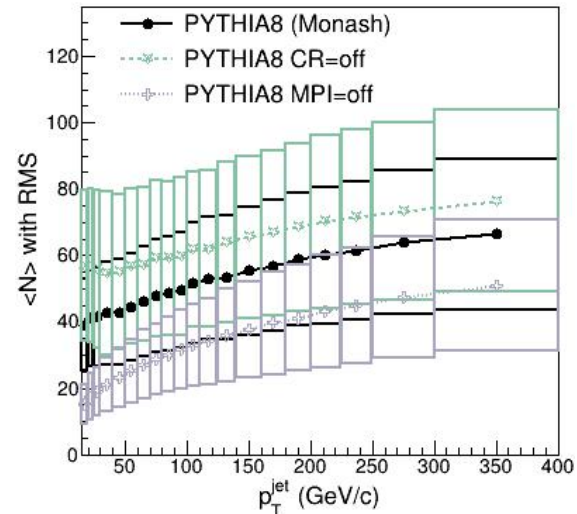
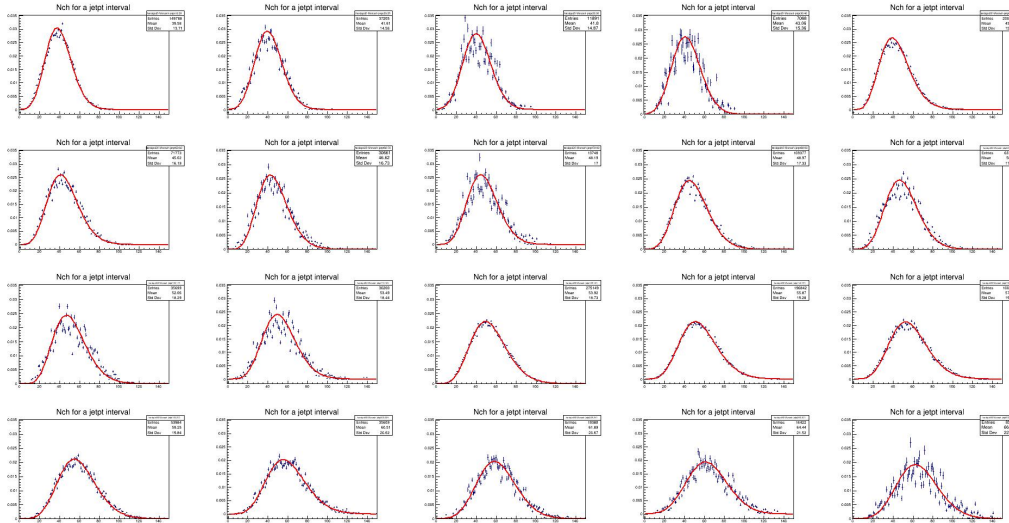


KNO within jet: multiplicity scaling with p_T^{jet}



- Multiplicity (dominated by the jet multiplicity) vs. jet momentum p_T^{jet}

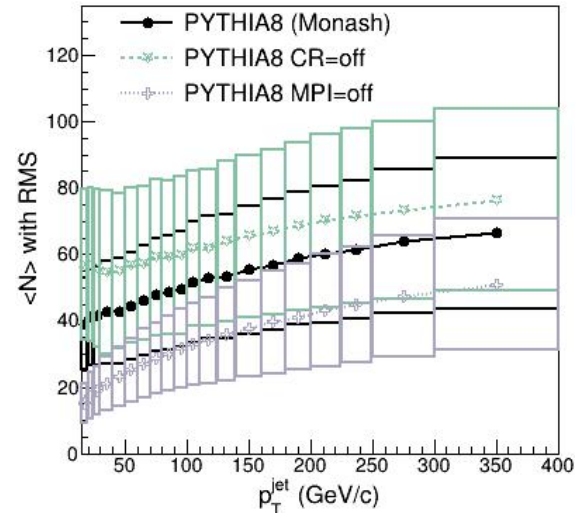
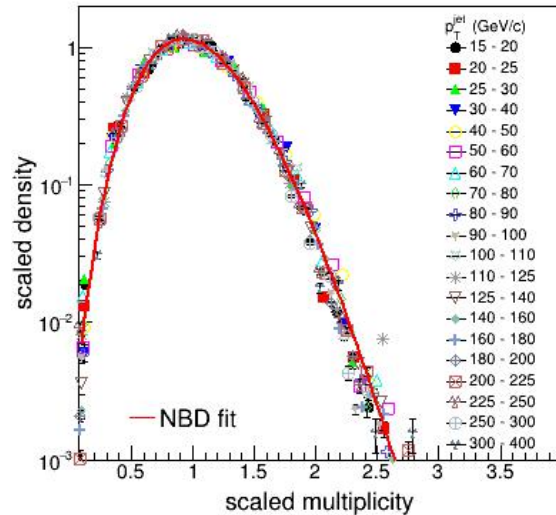
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KNO within jet: multiplicity scaling with p_T^{jet}



- Multiplicity (dominated by the jet multiplicity) vs. jet momentum p_T^{jet}
- Parametrized with a NBD
- Distributions at all p_T^{jet} fit well on a single NBD curve
- **KNO-like scaling observed within a jet**
 - In the following we quantify how well it is fulfilled

$$P_N = \frac{\Gamma(Nk + a)}{\Gamma(a)\Gamma(Nk + 1)} p^{Nk} (1 - p)^a$$

Multiplicity vs. p_T^{jet} : moments

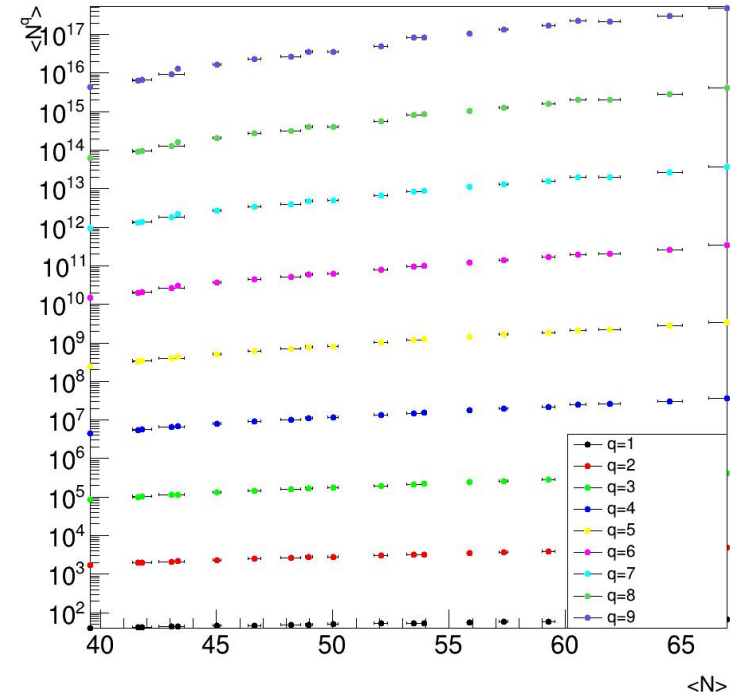
- q^{th} statistical moment

$$\langle N^q \rangle = \sum_{N=1}^{\infty} P_N N^q$$

- sensitive to goodness of scaling
- insensitive to fluctuations
- no need to parametrize and fit

- **Scaling:**

$$\langle N^q(p_T^{\text{jet}}) \rangle = \lambda^q(p_T^{\text{jet}}) \langle N^q(p_0) \rangle \quad \lambda(p_0) = 1$$



Multiplicity vs. p_T^{jet} : moments

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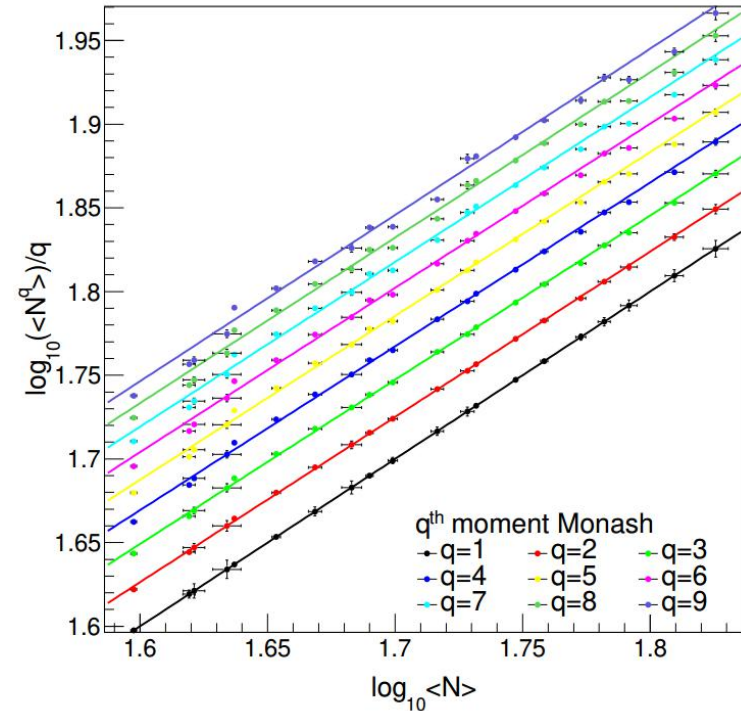
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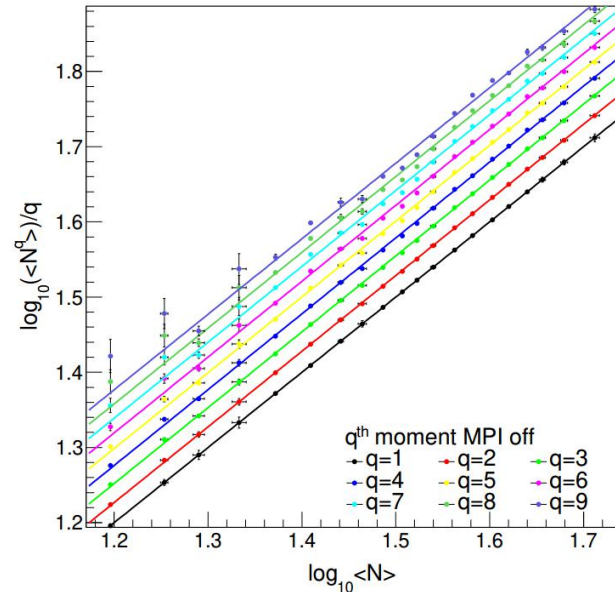
$$\langle N^q(p_T^{\text{jet}}) \rangle = \lambda^q(p_T^{\text{jet}}) \langle N^q(p_0) \rangle \quad \lambda(p_0) = 1$$

- $\log \langle N^q \rangle / q$ vs. $\log \langle N \rangle$ is a straight line with \sim unity slope
 - up to the 9th moment
- => scaling is fulfilled in the whole p_T^{jet} range**

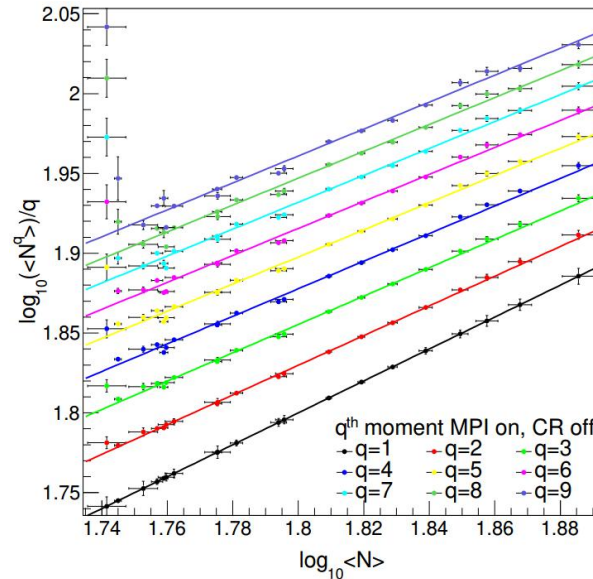


Moments: Role of MPI and CR

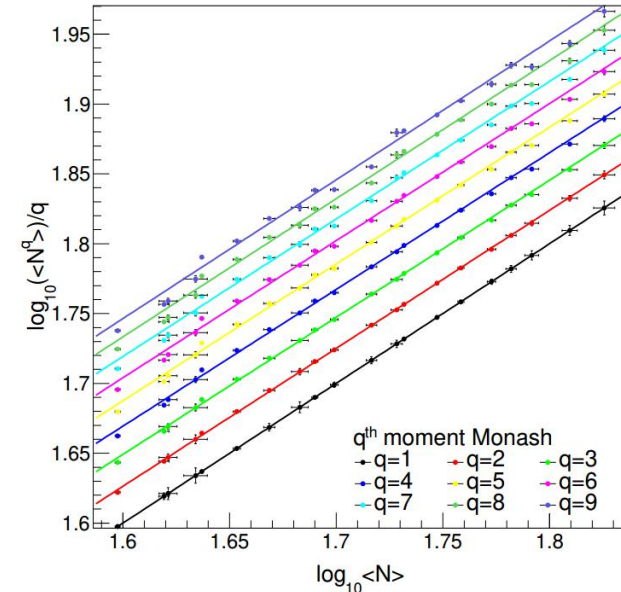
MPI off



CR off, MPI on



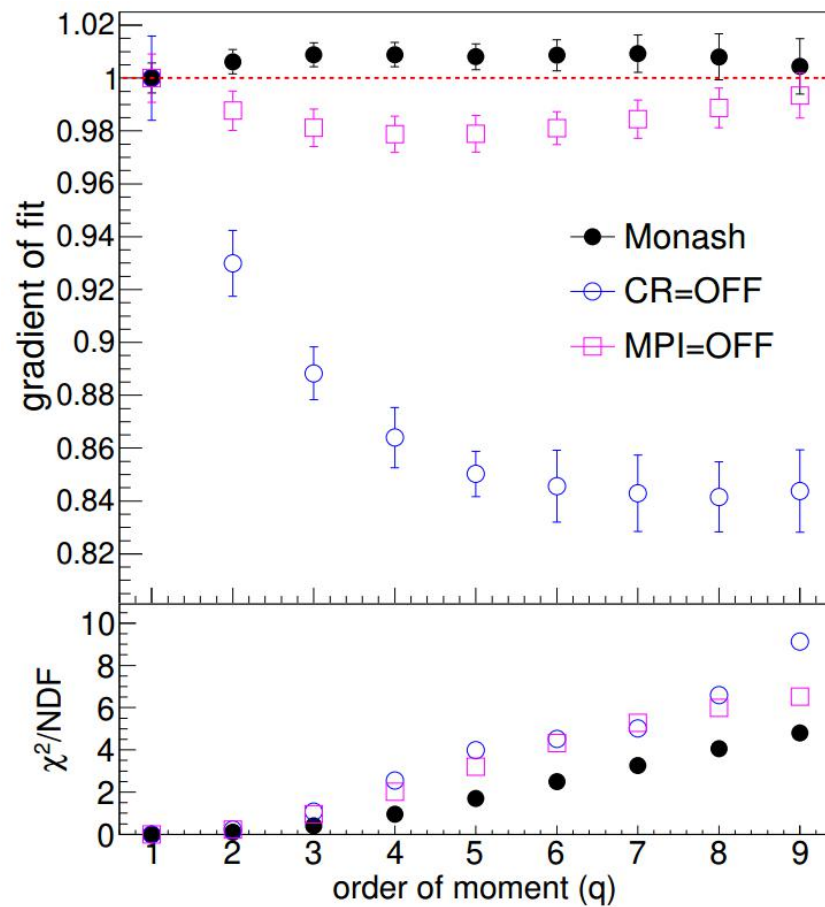
physical: CR on, MPI on



- **No multiple-parton interactions: scaling is present**
 - “possible physical” scenario producing low-activity events
- **No color reconnection: no scaling**
 - color-flow not handled, non-physical scenario

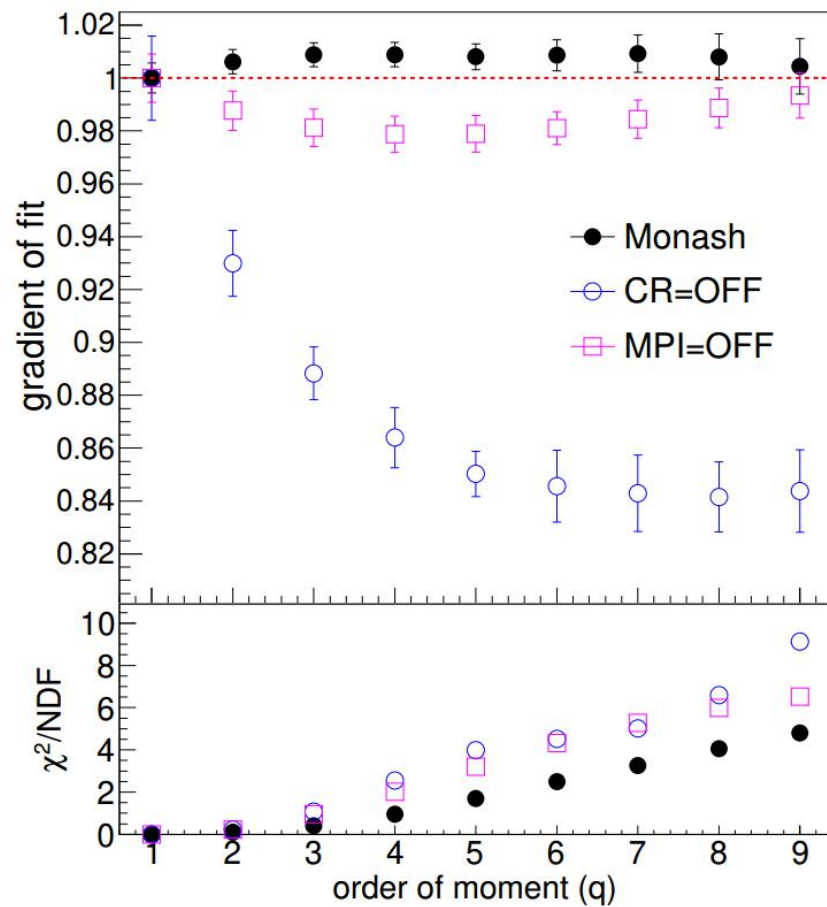
Slopes moment-by-moment

- Physical case (Monash): All 9 moments are consistent with unity, slope within $\sim 1\%$
 - Note*: scaling holds for different tunes & nPDFs (Monash, 4C, Monash*) and also for different jet algos (anti- k_T , C/A and k_T)
- No CR: Scaling is broken by $\sim 15\%$
- No MPI (also no CR by construction): Scaling is fulfilled to $\sim 2\%$.
- All fits are statistically good ($\chi^2/\text{NDF} < 8$, \sim proportional to the order of moment)



Slopes moment-by-moment

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 - Note*: scaling holds for different tunes & nPDFs (Monash, 4C, Monash*) and also for different jet algos (anti- k_T , C/A and k_T)
- No CR: Scaling is broken by $\sim 15\%$
- No MPI (also no CR by construction): Scaling is fulfilled to $\sim 2\%$.
- All fits are statistically good ($\chi^2/\text{NDF} < 8$, \sim proportional to the order of moment)
- The emerging picture is different from that of radial profile scaling, which holds for CR=off as well**



Summary

- We observed scaling behavior in jets from 7 TeV pp collisions using MC
- **Radial jet-momentum profiles scale with multiplicity**
 - Profiles can be parametrized with a Poissonian, and scale with event multiplicity
 - Scaling is present in a broad model class, regardless of settings (nPDF, CR, MPI settings, jet reconstruction, and even MC generator)
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 - Multiplicity distributions are NBD and can be collapsed into a single distribution
 - This scaling holds without MPI but breaks down without CR
 - **KNO scaling is likely violated by complex QCD processes outside the jet development, such as single and double-parton scatterings or softer MPI**
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Thank you!

**Special thanks to Sándor Hegyi
for fruitful discussions and guidance**

Scaling of the jet profiles - log scale

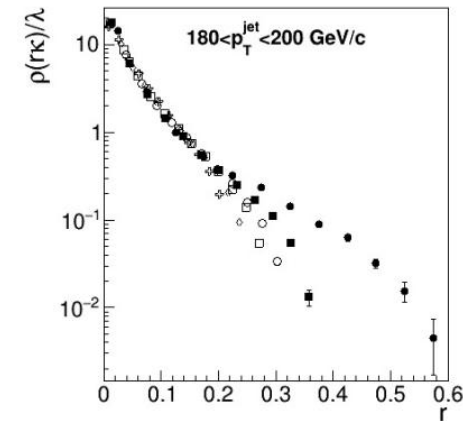
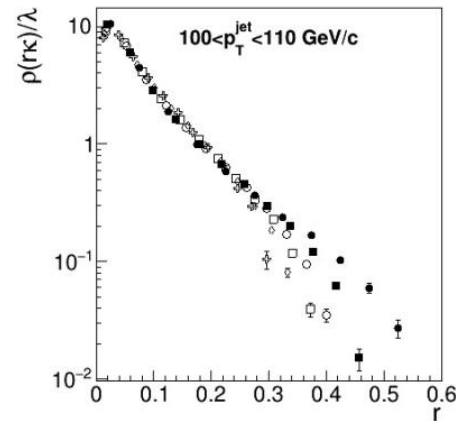
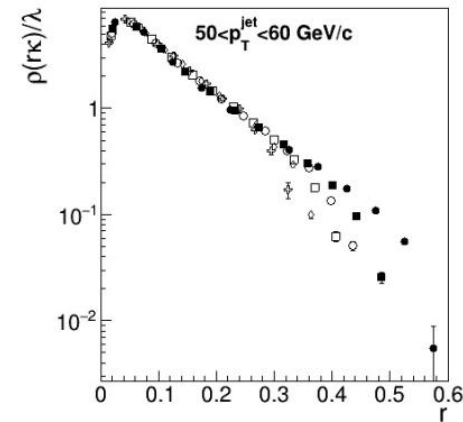
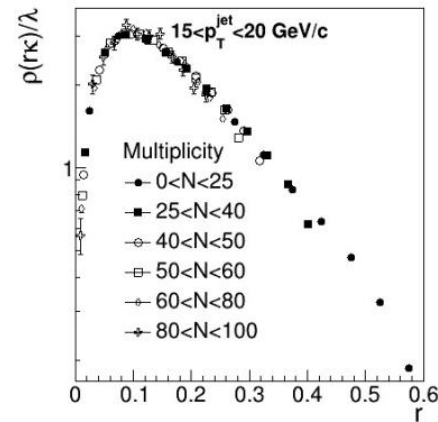
- Scaling assumption: profiles at all multiplicities collapse into a single distribution,

$$\rho_N(r) = \lambda(N) f\left(\frac{r}{\kappa(N)}\right)$$

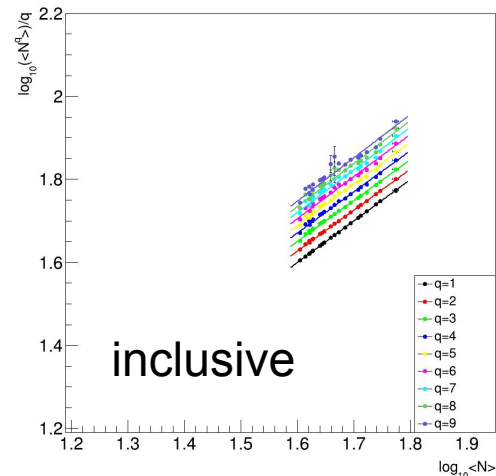
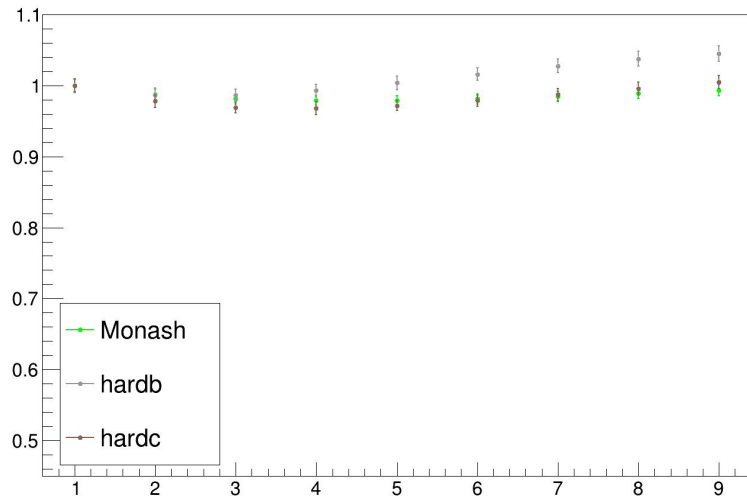
Note: Ideally, $\lambda=1/\kappa$, however...

“leakage” (distribution is cut-off at high r before normalization)

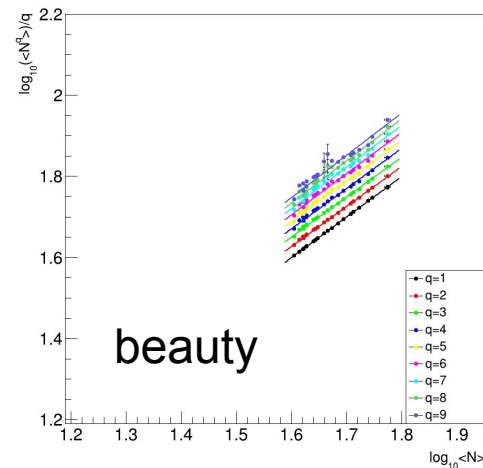
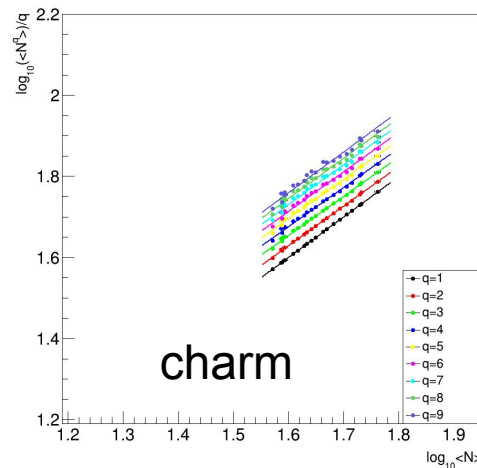
- Scaling is determined based on Poissonian fits
 - Chosen “good” mid-multiplicity fits, then others scaled to it minimizing χ^2
- The scaling works within 5-10% in the peak region**



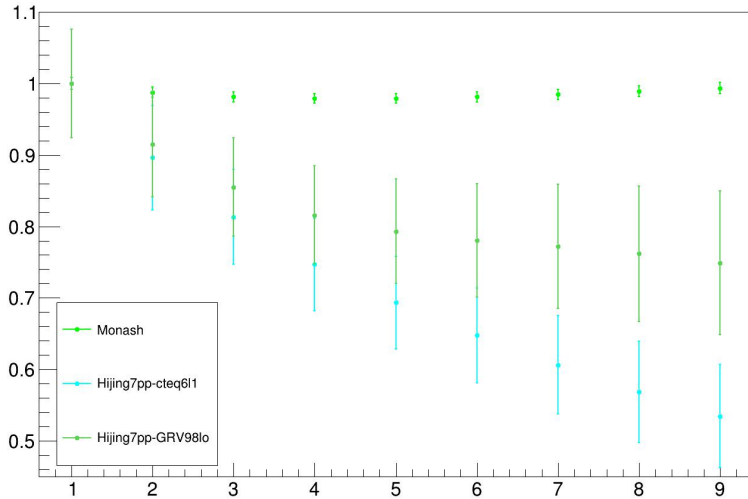
KNO-like scaling: Heavy Flavor



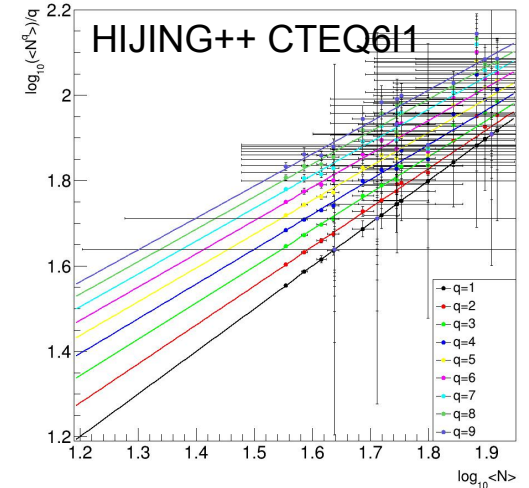
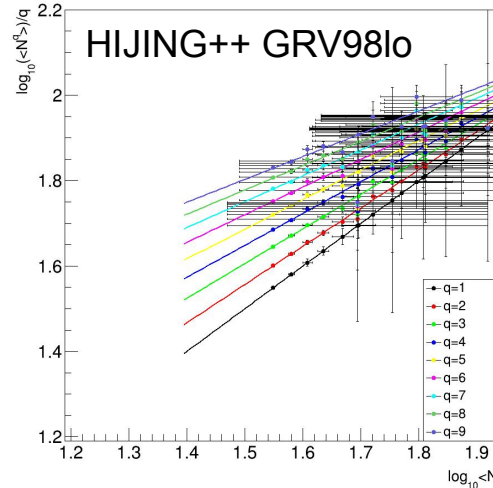
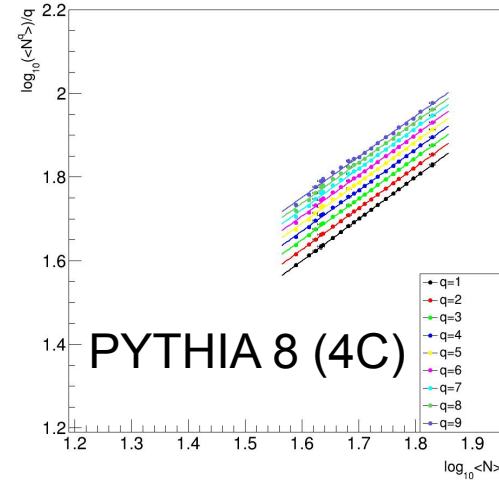
- Heavy-flavor jets also show KNO-like scaling



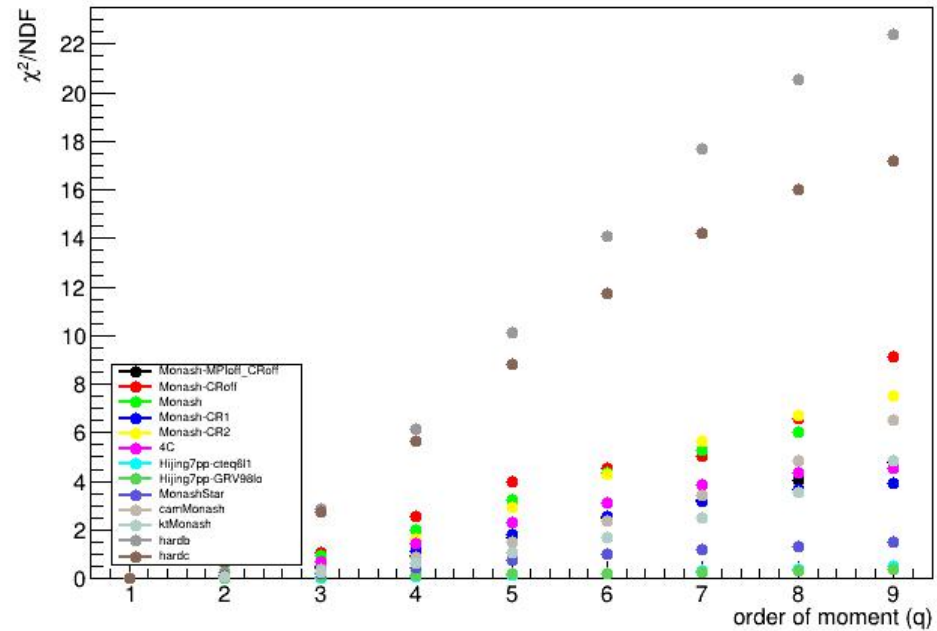
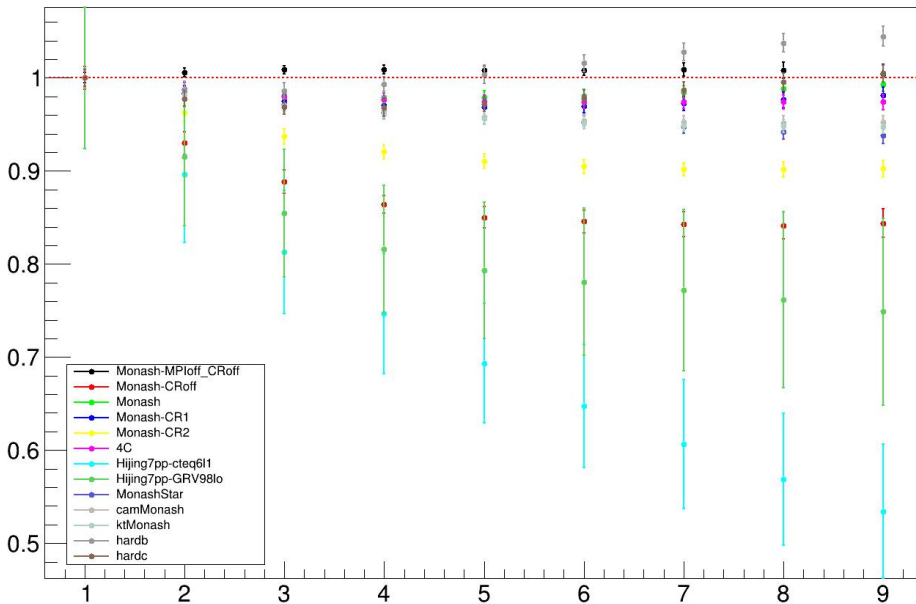
KNO-like scaling: Hijing++



- Hijing++ does not exhibit the scaling



KNO-like scaling: summary

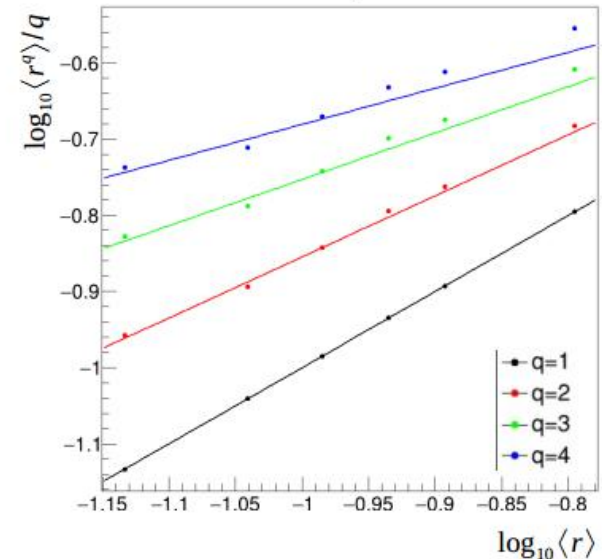
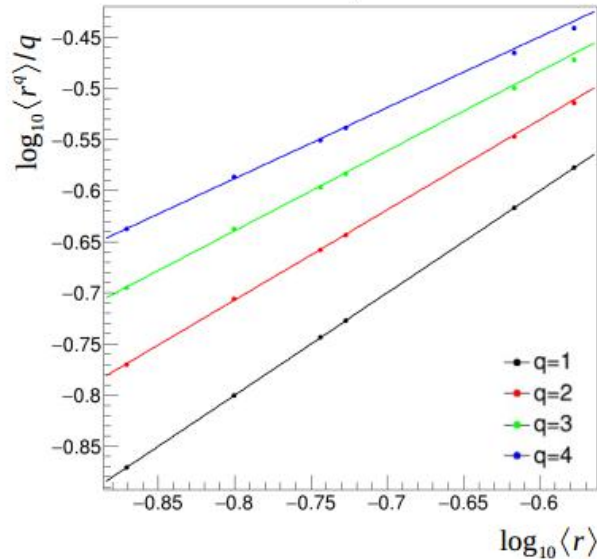
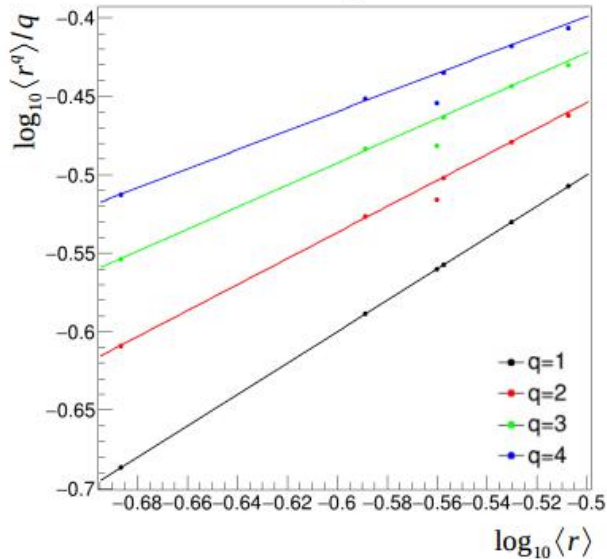


Statistical moments of jet profiles (Monash with MPI and CR)

$15 < p_T < 20$

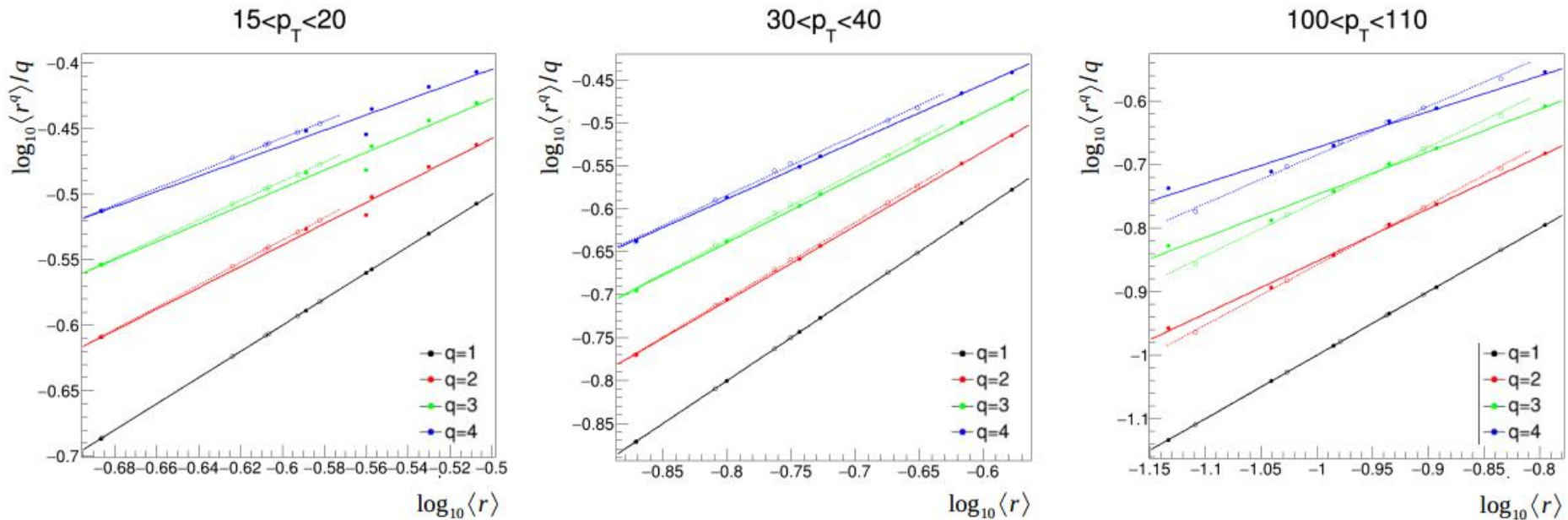
$30 < p_T < 40$

$100 < p_T < 110$



The gradients are not 1, but it could be explained with the binning.

Effects of finite-size bins (jet profiles)



Dotted lines: effect of binning on analytical curves.
Qualitatively explains the behavior seen in the simulations.